Microwave Engineering Course

4th Stage

Electrical Engineering Department

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Instructor

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Part VI

Microwave Components
Outline course

- Resonant cavities
- Rectangular cavity resonators
- Circular cavity resonators
- Q-factor of cavity resonator
- Microwave hybrid circuits
- The S-parameter theory
- Properties of S-parameter
- Three port devices
- Directional couplers
Outline course

- Circulators and isolators
- Hybrid couplers
Outline course

- Resonant cavities
  - Rectangular cavity resonators
  - Circular cavity resonators
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Microwave resonators are used in a variety of applications, including filters, oscillators, frequency meters, and tuned amplifiers. Because the operation of microwave resonators is very similar to that of lumped-element resonators of circuit theory, we will begin by reviewing the basic characteristics of series and parallel $RLC$ resonant circuits. We will then discuss various implementations of resonators at microwave frequencies using distributed elements such as transmission lines, rectangular and circular waveguides, and dielectric cavities. We will also discuss the excitation of resonators using apertures and current sheets.
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- Resonant cavities
- **Rectangular cavity resonators**
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Microwave resonators can also be constructed from closed sections of waveguide. Because radiation loss from an open-ended waveguide can be significant, waveguide resonators are usually short circuited at both ends, thus forming a closed box, or cavity. Electric and magnetic energy is stored within the cavity enclosure, and power is dissipated in the metallic walls of the cavity as well as in the dielectric material that may fill the cavity. Coupling to a cavity resonator may be by a small aperture, or a small probe or loop. We will see that there are many possible resonant modes for a cavity resonator, corresponding to field variations along the three dimensions of the structure.

We will first derive the resonant frequencies for a general TE or TM resonant mode of a rectangular cavity, and then derive an expression for the unloaded $Q$ of the $\text{TE}_{101}$ mode. A complete treatment of the unloaded $Q$ for arbitrary TE and TM modes can be made using the same procedure, but is not included here because of its length and complexity.

**Resonant Frequencies**

The geometry of a rectangular cavity is shown in Figure 6.6. It consists of a length, $d$, of rectangular waveguide shorted at both ends ($z = 0, d$). We will find the resonant
FIGURE 6.6  A rectangular cavity resonator, and the electric field variations for the $\text{TE}_{101}$ and $\text{TE}_{102}$ resonant modes.
frequencies of this cavity under the assumption that the cavity is lossless, then determine the unloaded $Q$ using the perturbation method outlined in Section 2.7. Although we could begin with the Helmholtz wave equation and the method of separation of variables to solve for the electric and magnetic fields that satisfy the boundary conditions of the cavity, it is easier to start with the fields of the TE or TM waveguide modes since these already satisfy the necessary boundary conditions on the side walls ($x = 0, a$ and $y = 0, b$) of the cavity. Then it is only necessary to enforce the boundary conditions that $E_x = E_y = 0$ on the end walls at $z = 0, d$. The transverse electric fields ($E_x$, $E_y$) of the $\text{TE}_{mn}$ or $\text{TM}_{mn}$ rectangular waveguide mode can be written as

$$\bar{E}_t(x, y, z) = \bar{e}(x, y)(A^+ e^{-j\beta_{mn}z} + A^- e^{+j\beta_{mn}z})$$
where $\tilde{e}(x, y)$ is the transverse variation of the mode, and $A^+, A^-$ are arbitrary amplitudes of the forward and backward traveling waves. The propagation constant of the $m, n$th TE or TM mode is

$$\beta_{mn} = \sqrt{k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

where $k = \omega \sqrt{\mu \varepsilon}$, $\varepsilon$ and $\mu$ are the permeability and permittivity of the material filling the cavity. Applying the condition that $\vec{E}_t = 0$ at $z = 0$ to (6.36) implies that $A^+ = -A^-$ (as we should expect for reflection from a perfectly conducting wall). Then the condition that $\vec{E}_t = 0$ at $z = d$ leads to the equation

$$\vec{E}_t = -\tilde{e}(x, y)A^+ 2j\sin\beta_{mn} d = 0$$

$$\beta_{mn} d = l\pi, \quad l = 1, 2, \ldots$$

A resonance wave number for the rectangular cavity can be defined as
\[ k_{mn\ell} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \]

Then we can refer to the TE\(_{mn\ell}\) or TM\(_{mn\ell}\) resonant mode of the cavity, where the indices \(m, n, \ell\) indicate the number of variations in the standing wave pattern in the \(x, y, z\) directions, respectively. The resonant frequency of the TE\(_{mn\ell}\) or TM\(_{mn\ell}\) mode is given by

\[ f_{mn\ell} = \frac{ck_{mn\ell}}{2\pi\sqrt{\mu_r\varepsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r\varepsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \]

If \(b<a<d\), the dominant resonant mode (lowest resonant frequency) will be the TE\(_{101}\) mode, corresponding to the TE\(_{10}\) dominant waveguide mode in a shorted guide of length \(\lambda_g/2\), and is similar to the short-circuited \(\lambda/2\) transmission line resonator. The dominant TM resonant mode is the TM\(_{110}\) mode.
The stored electric energy is,

\[ W_e = \frac{\varepsilon}{4} \int_{V} E_y \cdot E_y^* \, dv = \frac{\varepsilon}{4} \int_{0}^{a} \int_{0}^{b} \int_{0}^{d} E_o^2 \left( \sin \frac{\pi x}{a} \right)^2 \left( \sin \frac{\pi z}{d} \right)^2 \, dx \, dy \, dz \]

\[ = \frac{\varepsilon b E_o^2}{4} \int_{0}^{a} \frac{1}{2} \left( 1 - \cos \frac{2\pi x}{a} \right) \, dx \int_{0}^{d} \frac{1}{2} \left( 1 - \cos \frac{2\pi z}{d} \right) \, dz = \frac{\varepsilon abd E_o^2}{16} \]

while the stored magnetic energy is,

\[ W_m = \frac{\mu}{4} \int_{V} (H_x \cdot H_x^* + H_z \cdot H_z^*) \, dv = \frac{\mu abd E_o^2}{16} \left( \frac{1}{Z_{TE}^2} + \frac{\pi^2}{k^2 \eta^2 a^2} \right) = \frac{\mu abd E_o^2}{16 \eta^2} = W_e \]

Showing that \( W_e = W_m \) at resonance. The condition of equal stored electric and magnetic energies at resonance also applied to the \( RLC \) resonant circuits.
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CIRCULAR WAVEGUIDE CAVITY RESONATORS

A cylindrical cavity resonator can be constructed from a section of circular waveguide shorted at both ends, similar to rectangular cavities. Because the dominant circular waveguide mode is the TE_{11} mode, the dominant cylindrical cavity mode is the TE_{111} mode. We will derive the resonant frequencies for the TE_{nm} and TM_{nm} circular cavity modes, and an expression for the unloaded Q of the TE_{nm} mode. Circular cavities are often used for microwave frequency meters. The cavity is constructed with a movable top wall to allow mechanical tuning of the resonant frequency, and the cavity is loosely coupled to a waveguide through a small aperture. In operation, power will be absorbed by the cavity as it is tuned to the operating frequency of the system; this absorption can be monitored with a power meter elsewhere in the system.
The mechanical tuning dial is usually directly calibrated in frequency, as in the model shown in Figure 6.7. Because frequency resolution is determined by the $Q$ of the resonator, the TE011 mode is often used for frequency meters because its $Q$ is much higher than the $Q$ of the dominant circular cavity mode. This is also the reason for a loose coupling to the cavity.

Figure 6.7  Photograph of a W-band waveguide frequency meter. The knob rotates to change the length of the circular cavity resonator; the scale gives a readout of the frequency. Photograph courtesy of Millitech Inc., Northampton, Mass.
Resonant Frequencies

The geometry of a cylindrical cavity is shown in Figure 6.8. As in the case of the rectangular cavity, the solution is simplified by beginning with the circular waveguide modes, which already satisfy the necessary boundary conditions on the wall of the circular waveguide. From Table 3.5, the transverse electric fields \((E_\rho, E_\phi)\) of the TE\(_{nm}\) or TM\(_{nm}\) circular waveguide mode can be written as

\[
\bar{E}_t(\rho, \phi, z) = \bar{e}(\rho, \phi) \left( A^+ e^{-j\beta_{mn}z} + A^- e^{+j\beta_{mn}z} \right)
\]

where \(\bar{e}(\rho, \phi)\) represents the transverse variation of the mode, and \(A^+\) and \(A^-\) are arbitrary amplitudes of the forward and backward traveling waves. The propagation constant of the TE\(_{nm}\) mode is, from (3.126),

\[
\beta_{mn} = \sqrt{k^2 + \left( \frac{p'_{mn}}{a} \right)^2}
\]
while the propagation constant of the TM$_{nm}$ mode is, from (3.139),

$$\beta_{mn} = \sqrt{k^2 + \left( \frac{p_{mn}}{a} \right)^2}$$

where $k = \omega \sqrt{\mu \varepsilon}$. 

**FIGURE 6.8** A cylindrical resonant cavity, and the electric field distribution for resonant modes with $\ell = 1$ or $\ell = 2$. 

\[ d \]

\[ \ell = 2 \]

\[ \ell = 1 \]

\[ E_\rho E_\phi \]
In order to have $\vec{E}_t = 0$ at $z = 0, d$, we must choose $A^+ = -A^-$, and $A^+ \sin \beta_{nm} d = 0$,

or

$$\beta_{nm} d = \ell \pi, \quad \text{for } \ell = 0, 1, 2, 3, \ldots,$$

which implies that the waveguide must be an integer number of half-guide wavelengths long. Thus, the resonant frequency of the $\text{TE}_{nm\ell}$ mode is

$$f_{nm\ell} = \frac{c}{2\pi \sqrt{\mu_r \varepsilon_r}} \sqrt{\left( \frac{p'_{nm}}{a} \right)^2 + \left( \frac{\ell \pi}{d} \right)^2}, \quad (6.53a)$$

and the resonant frequency of the $\text{TM}_{nm\ell}$ mode is

$$f_{nm\ell} = \frac{c}{2\pi \sqrt{\mu_r \varepsilon_r}} \sqrt{\left( \frac{p_{nm}}{a} \right)^2 + \left( \frac{\ell \pi}{d} \right)^2}. \quad (6.53b)$$

Thus the dominant TE mode is the $\text{TE}_{111}$ mode, while the dominant TM mode is the $\text{TM}_{010}$ mode. Figure 6.9 shows a mode chart for the lower order resonant modes of a cylindrical cavity. Such a chart is useful for the design of circular cavity resonators, as it shows what modes can be excited at a given frequency for a given cavity size.
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Quality factor: Is the ratio of stored energy to energy dissipated per cycle divided by $2\pi$

Quality Factor $Q_0$:

$$Q_0 = \frac{\text{Energy stored in cavity}}{\text{Energy dissipated in cavity walls per radian}} = \frac{\omega_0 U}{P_{\text{diss}}}$$

$$= \omega_0 \tau_0 = \frac{\omega_0}{\Delta \omega_0}$$

Lower surface resistance gives higher Q. For a given R/Q, this gives higher R. Lower external power is required for a given voltage V.
Stored energy over cavity volume is

\[
W_s = \frac{\varepsilon_0}{2} \int |E|^2 dv
\]

where the first integral applies to the time the energy is stored in the E-field and the second integral as it oscillates back into the H-field.

Losses on cavity walls are introduced by taking into account the finite conductivity \( \sigma \) of the walls.

Since, for a perfect conductor, the linear density of the current \( \mathbf{J} \) along walls of structure is

\[
\mathbf{J} = \hat{n} \times \mathbf{H}
\]
We can write

\[ P_d = \frac{R_{surf}}{2} \int_s |H|^2 ds \]

with \( s \) = inner surface of conductor

\( R_{surf} \) = surface resistance

\( \delta \) = skin depth

\[ R_{surf} = \sqrt{\frac{\pi f \mu_0 \mu \tau}{\sigma}} = \frac{1}{\sigma \delta} \]

For Cu, \( R_{surf} = 2.61 \times 10^{-7} \sqrt{\omega} \ \Omega \)
A. Q- factor of rectangular cavity resonators

The unloaded $Q$ of the cavity with lossy conducting walls but lossless dielectric can be found as

- For small losses we can find the power dissipated in the cavity walls using the perturbation method of Section 2.7. Thus, the power lost in the conducting walls is given by (1.131) as

$$P_c = \frac{R_s}{2} \int_{wall} |H_t|^2 ds$$

Where $R = \sqrt{\omega \mu_0 \nu \sigma}$ is the surface resistivity of the metallic walls, and $H_t$ is the tangential magnetic field at the surface of the walls. Using (6.42b), (6.42c) in (6.44) gives
Unloaded Q of the TE10 Mode

From Table 3.2, (6.36), and the fact that $A^- = -A^+$, the total fields for the TE10 resonant mode can be written as

\[
E_y = A^+ \sin \frac{\pi x}{a} \left( e^{-j\beta z} - e^{j\beta z} \right),
\]

\[
H_x = \frac{-A^+}{Z_{TE}} \sin \frac{\pi x}{a} \left( e^{-j\beta z} + e^{j\beta z} \right),
\]

\[
H_z = \frac{j \pi A^+}{k \eta a} \cos \frac{\pi x}{a} \left( e^{-j\beta z} - e^{j\beta z} \right).
\]

Letting $E_0 = -2jA^+$ and using (6.38) allows these expressions to be simplified to

\[
E_y = E_0 \sin \frac{\pi x}{a} \sin \frac{\ell \pi z}{d},
\]

\[
H_x = \frac{-jE_0}{Z_{TE}} \sin \frac{\pi x}{a} \cos \frac{\ell \pi z}{d},
\]

\[
H_z = \frac{j \pi E_0}{k \eta a} \cos \frac{\pi x}{a} \sin \frac{\ell \pi z}{d}.
\]
\[
P_c = \frac{R_s}{2} \left\{ 2 \int_0^b \int_0^a |H_x(z = 0)|^2 \, dx \, dy + 2 \int_0^d \int_0^b |H_z(x = 0)|^2 \, dy \, dz + \right. \\
\left. 2 \int_0^d \int_0^a [\, |H_x(y = 0)|^2 + |H_z(x = 0)|^2 \, ] \, dx \, dz \\
= \frac{R_s E_0^2 \lambda^2}{8 \eta^2} \left( \frac{l^2 ab}{d^2} + \frac{bd}{a^2} + \frac{l^2 a}{2d} + \frac{d}{2a} \right) \right\}
\]

where use has been made of the symmetry of the cavity in doubling the contributions from the walls at \( x = 0, y = 0, \) and \( z = 0 \) to account for the contributions from the walls at \( x = a, y = b, \) and \( z = d, \) respectively. The relations \( k = 2 \pi / \lambda \) and \( Z_{TE} = k \eta / \beta = 2d \eta / \lambda \) were also used in simplifying (6.45). Then, from (6.7), the unloaded \( Q \) of the cavity with lossy conducting walls but lossless dielectric can be found as:

\[
H_x = -\frac{j E_0}{Z_{TE}} \sin \frac{\pi x}{a} \cos \frac{\ell \pi z}{d}, \\
H_z = \frac{j \pi E_0}{k \eta} \cos \frac{\pi x}{a} \sin \frac{\ell \pi z}{d},
\]
\[ Q_c = \frac{2\omega_0 W_e}{P_c} \]
\[ = \frac{k^3 abd\eta}{4\pi^2 R_s} \frac{1}{[(\ell^2 ab/d^2) + (bd/a^2) + (\ell^2 a/2d) + (d/2a)]} \]
\[ = \frac{(kad)^3 b\eta}{2\pi^2 R_s} \frac{1}{(2\ell^2 a^3 b + 2bd^3 + \ell^2 a^3 d + ad^3)}. \]  

(6.46)

Next we compute the power lost in the dielectric material that may fill the cavity. As discussed in Chapter 1, a lossy dielectric has an effective conductivity \( \sigma = \omega \varepsilon'' = \omega \varepsilon_r \varepsilon_0 \tan \delta \), where \( \varepsilon = \varepsilon' - j \varepsilon'' = \varepsilon_r \varepsilon_0 (1 - j \tan \delta) \), and \( \tan \delta \) is the loss tangent of the material. The power dissipated in the dielectric is, from (1.92),

\[ P_d = \frac{1}{2} \int_V \vec{J} \cdot \vec{E}^* dv = \frac{\omega \varepsilon''}{2} \int_V |\vec{E}|^2 dv = \frac{abdw_0 \varepsilon'' |E_0|^2}{8}, \]  

(6.47)

where \( \vec{E} \) is given by (6.42a). Then from (6.7) the unloaded \( Q \) of the cavity with a lossy dielectric filling, but with perfectly conducting walls, is

\[ Q_d = \frac{2\omega W_e}{P_d} = \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan \delta}. \]  

(6.48)

The simplicity of this result is due to the fact that the integral in (6.43a) for \( W_e \) cancels with the identical integral in (6.47) for \( P_d \). This result therefore applies to \( Q_d \) for an arbitrary resonant cavity mode. When both wall losses and dielectric losses are present, the total power loss is \( P_c + P_d \), so (6.7) gives the total unloaded \( Q \) as

\[ Q_0 = \left( \frac{1}{Q_c} + \frac{1}{Q_d} \right)^{-1}. \]  

(6.49)
EXAMPLE 6.3  DESIGN OF A RECTANGULAR CAVITY RESONATOR

A rectangular waveguide cavity is made from a piece of copper WR-187 H-band waveguide, with $a = 4.755$ cm and $b = 2.215$ cm. The cavity is filled with polyethylene ($\varepsilon_r = 2.25$, $\tan\delta = 0.0004$). If resonance is to occur at $f = 5$ GHz, find the required length, $d$, and the resulting unloaded $Q$ for the $\ell = 1$ and $\ell = 2$ resonant modes.

\[
\begin{align*}
\text{a} &= 4.755 \text{ cm} \\
\text{b} &= 2.215 \text{ cm} \\
\text{d} &= ?
\end{align*}
\]

H band is 6-8 GHz
Solution

The wave number \( k \) is

\[
k = \frac{2\pi f \sqrt{\varepsilon_r}}{c} = 157.08 \text{ m}^{-1}.
\]

From (6.40) the required length for resonance can be found as \((m = 1, n = 0)\)

\[
d = \frac{\ell \pi}{\sqrt{k^2 - (\pi/a)^2}},
\]

for \( \ell = 1, \quad d = \frac{\pi}{\sqrt{(157.08)^2 - (\pi/0.04755)^2}} = 2.20 \text{ cm},
\]

for \( \ell = 2, \quad d = 2(2.20) = 4.40 \text{ cm}.
\]

From Example 6.1, the surface resistivity of copper at 5 GHz is \( R_s = 1.84 \times 10^{-2} \Omega \). The intrinsic impedance is

\[
\eta = \frac{377}{\sqrt{\varepsilon_r}} = 251.3 \Omega.
\]

Then from (6.46) the \( Q \) due to conductor loss only is

for \( \ell = 1, \quad Q_c = 8,403, \]

for \( \ell = 2, \quad Q_c = 11,898. \]

From (6.48) the \( Q \) due to dielectric loss only is, for both \( \ell = 1 \) and \( \ell = 2, \)

\[
Q_d = \frac{1}{\tan \delta} = \frac{1}{0.0004} = 2500.
\]
Then total unloaded $Q$s are, from (6.49)

for $\ell = 1$, \[ Q_0 = \left( \frac{1}{8403} + \frac{1}{2500} \right)^{-1} = 1927, \]

for $\ell = 2$, \[ Q_0 = \left( \frac{1}{11,898} + \frac{1}{2500} \right)^{-1} = 2065. \]

Note that the dielectric loss has the dominant effect on the $Q$; higher $Q$ could be obtained using an air-filled cavity. These results can be compared to those of Examples 6.1 and 6.2, which used similar types of materials at the same frequency.
B. Q-factor of circular cavity resonators

The power loss in the conducting walls is

\[ P_c = \frac{R_s}{2} \int_S |\vec{H}_{\text{tan}}|^2 ds \]

\[ = \frac{R_s}{2} \left\{ \int_{z=0}^{d} \int_{\phi=0}^{2\pi} \left[ |H_\phi(\rho = a)|^2 + |H_z(\rho = a)|^2 \right] a d\phi dz \right. \]

\[ + 2 \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \left[ |H_\rho(z = 0)|^2 + |H_\phi(z = 0)|^2 \right] \rho d\rho d\phi \right\} \]

\[ = \frac{R_s}{2} \pi H_0^2 J_n^2 (p'_{nm}) \left\{ \frac{da}{2} \left[ 1 + \left( \frac{\beta an}{(p'_{nm})^2} \right)^2 \right] + \left( \frac{\beta a^2}{p'_{nm}} \right)^2 \left( 1 - \frac{n^2}{(p'_{nm})^2} \right) \right\}. \quad (6.56) \]

Then, from (6.8), the unloaded \( Q \) of the cavity with imperfectly conducting walls but lossless dielectric is

\[ Q_c = \frac{\omega_0 W}{P_c} = \frac{(ka)^3 \eta ad}{4 (p'_{nm})^2 R_s} \left\{ \frac{1}{a d} \left[ 1 + \left( \frac{\beta a n}{(p'_{nm})^2} \right)^2 \right] + \left( \frac{\beta a^2}{p'_{nm}} \right)^2 \left( 1 - \frac{n^2}{(p'_{nm})^2} \right) \right\}. \quad (6.57) \]
From (6.52) and (6.51) we see that $\beta = \ell \pi/d$ and $(ka)^2$ are constants that do not vary with frequency, for a cavity with fixed dimensions. Thus, the frequency dependence of $Q_c$ is given by $k/R_5$, which varies as $1/\sqrt{f}$; this gives the variation in $Q_c$ for a given resonant mode and cavity shape (fixed $n, m, \ell$, and $a/d$).

Figure 6.10 shows the normalized unloaded $Q$ due to conductor loss for various resonant modes of a cylindrical cavity. Observe that the TE$_{011}$ mode has an unloaded $Q$ significantly higher than that of the lower order TE$_{111}$, TM$_{010}$, or TM$_{111}$ mode.

To compute the unloaded $Q$ due to dielectric loss, we must compute the power dissipated in the dielectric. Thus,

$$P_d = \frac{1}{2} \int_V \mathbf{J} \cdot \mathbf{E}^* dV = \frac{\omega \epsilon''}{2} \int_V \left[ |E_{\rho}|^2 + |E_{\phi}|^2 \right] dV$$

$$= \frac{\omega \epsilon'' k^2 \eta^2 a^2 H_0^2 \pi d}{4(p_{nm}')^2} \int_{\rho=0}^{a} \left[ \left( \frac{na}{p_{nm}' \rho} \right)^2 J_n^2 \left( \frac{p_{nm}' \rho}{a} \right) + J_n^2 \left( \frac{p_{nm}' \rho}{a} \right) \right] \rho d\rho$$

$$= \frac{\omega \epsilon'' k^2 \eta^2 a^4 H_0^2}{8(p_{nm}')^2} \left[ 1 - \left( \frac{n}{p_{nm}'} \right)^2 \right] J_n^2(p_{nm}'). \tag{6.58}$$

Then (6.8) gives the unloaded $Q$ due to dielectric loss as

$$Q_d = \frac{\omega W}{P_d} = \frac{\epsilon}{\epsilon''} = \frac{1}{\tan \delta}, \tag{6.59}$$

where $\tan \delta$ is the loss tangent of the dielectric. This is the same as the result for $Q_d$ of (6.48) for the rectangular cavity. When both conductor and dielectric losses are present, the total unloaded cavity $Q$ can be found from (6.49).
FIGURE 6.10  Normalized unloaded $Q$ for various cylindrical cavity modes (air filled).
Unloaded Q of the TE\textsubscript{nm\ell} Mode

From Table 3.5, (6.50), and the fact that \( A^+ = - A^- \), the fields of the TE\textsubscript{nm\ell} mode can be written as

\[ H_z = H_0 J_n \left( \frac{p'_{nm \rho}}{a} \right) \cos n\phi \sin \frac{\ell \pi z}{d}, \]  
\[ (6.54a) \]

\[ H_\rho = \frac{\beta a H_0}{p'_{nm}} J'_n \left( \frac{p'_{nm \rho}}{a} \right) \cos n\phi \cos \frac{\ell \pi z}{d}, \]  
\[ (6.54b) \]

\[ H_\phi = \frac{- \beta a^2 n H_0}{(p'_{nm})^2 \rho} J_n \left( \frac{p'_{nm \rho}}{a} \right) \sin n\phi \cos \frac{\ell \pi z}{d}, \]  
\[ (6.54c) \]

\[ E_\rho = \frac{j k \eta a^2 n H_0}{(p'_{nm})^2 \rho} J_n \left( \frac{p'_{nm \rho}}{a} \right) \sin n\phi \sin \frac{\ell \pi z}{d}, \]  
\[ (6.54d) \]

\[ E_\phi = \frac{j k \eta a H_0}{p'_{nm}} J'_n \left( \frac{p'_{nm \rho}}{a} \right) \cos n\phi \sin \frac{\ell \pi z}{d}, \]  
\[ (6.54e) \]

\[ E_z = 0, \]  
\[ (6.54f) \]

where \( \eta = \sqrt{\mu/\varepsilon} \) and \( H_0 = -2j A^+ \).
Because the time-average stored electric and magnetic energies are equal, the total stored energy is

\[
W = 2W_e = \frac{\varepsilon}{2} \int_{z=0}^{d} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \left( |E_\rho|^2 + |E_\phi|^2 \right) \rho d\rho d\phi dz
\]

\[
= \frac{\varepsilon k^2 \eta^2 a^2 \pi d H_0^2}{4(p'_{nm})^2} \int_{\rho=0}^{a} \left[ J_n^2 \left( \frac{p'_{nm} \rho}{a} \right) + \left( \frac{na}{p'_{nm} \rho} \right)^2 J_n^2 \left( \frac{p'_{nm} \rho}{a} \right) \right] \rho d\rho
\]

\[
= \frac{\varepsilon k^2 \eta^2 a^4 H_0^2 \pi d}{8(p'_{nm})^2} \left[ 1 - \left( \frac{n}{p'_{nm}} \right)^2 \right] J_n^2(p'_{nm}),
\]  
(6.55)
EXAMPLE 6.4 DESIGN OF A CIRCULAR CAVITY RESONATOR

A circular cavity resonator with $d = 2a$ is to be designed to resonate at 5.0 GHz in the TE$_{011}$ mode. If the cavity is made from copper and is Teflon filled ($\varepsilon_r = 2.08$, $\tan \delta = 0.0004$), find its dimensions and unloaded $Q$. 
Solution

\[ k = \frac{2\pi f_{011} \sqrt{\varepsilon_r}}{c} = \frac{2\pi (5 \times 10^9) \sqrt{2.08}}{3 \times 10^8} = 151.0 \, \text{m}^{-1} \]

From (6.53a) the resonant frequency of the TE\textsubscript{011} mode is

\[ f_{011} = \frac{c}{2\pi \sqrt{\varepsilon_r}} \sqrt{\left( \frac{p'_{01}}{a} \right)^2 + \left( \frac{\pi}{d} \right)^2}, \]

with \( p'_{01} = 3.832 \). Then, since \( d = 2a \)

\[ \frac{2\pi f_{011} \sqrt{\varepsilon_r}}{c} = k = \sqrt{\left( \frac{p'_{01}}{a} \right)^2 + \left( \frac{\pi}{d} \right)^2}. \]

Solving for \( a \) gives

\[ a = \frac{\sqrt{(p'_{01})^2 + (\pi/2)^2}}{k} = \frac{\sqrt{(3.832)^2 + (\pi/2)^2}}{151.0} = 2.74 \, \text{cm}, \]

so we have \( d = 5.48 \, \text{cm} \).

The surface resistivity of copper at 5 GHz is \( R_s = 0.0184 \, \Omega \). Then from (6.57), with \( n = 0, m = \ell = 1, \) and \( d = 2a \), the unloaded \( Q \) due to conductor losses is

\[ Q_c = \frac{(ka)^3 \eta a d}{4 (p'_{01})^2 R_s \left[ ad/2 + (\beta a^2/p'_{01})^2 \right]} = \frac{kan \eta}{2 R_s} = 29,390, \]
where (6.51a) was used to simplify the expression. From (6.59) the unloaded $Q$ due to dielectric loss is

$$Q_d = \frac{1}{\tan \delta} = \frac{1}{0.0004} = 2500,$$

and the total unloaded $Q$ of the cavity is

$$Q_0 = \left( \frac{1}{Q_c} + \frac{1}{Q_d} \right)^{-1} = 2300.$$

This result can be compared with the rectangular cavity case of Example 6.3, which had $Q_0 = 1927$ for the $\text{TE}_{101}$ mode and $Q_0 = 2065$ for the $\text{TE}_{102}$ mode. If this cavity were air filled, the $Q$ would increase to 42,400.
Outline course

- Resonant cavities
- Rectangular cavity resonators
- Circular cavity resonators
- Q-factor of cavity resonator
- Microwave hybrid circuits
  - The S-parameter theory
  - Properties of S-parameter
  - Three point devices
  - Directional couplers
Microwave Hybrid Circuits

Microwave circuits consists of several microwave devices connected in some way to achieve the desired transmission of a microwave signal. The interconnection of two or more microwave devices may be regarded as a microwave junction. Such as waveguide Tees as the E-plane tee, H-plane tee, Magic tee, hybrid ring tee (rat-race circuit), directional coupler and the circulator.
(a) $E$-plane tee

(b) $H$-plane tee
(c) Magic tee

(d) Hybrid ring
(e) Directional couple:

(f) Circulator
Outline course

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S-parameters are a useful method for representing a circuit as a “black box”

The external behaviour of this black box can be predicted without any regard for the contents of the black box.

This black box could contain anything:

- a resistor,
- a transmission line
- or an integrated circuit.
A “black box” or network may have any number of ports.

This diagram shows a simple network with just 2 ports.

Note:

A port is a terminal pair of lines.
S-parameters are measured by sending a single frequency signal into the network or “black box” and detecting what waves exit from each port.

Power, voltage and current can be considered to be in the form of waves travelling in both directions.

For a wave incident on Port 1, some part of this signal reflects back out of that port and some portion of the signal exits other ports.

For high frequencies, it is convenient to describe a given network in terms of waves rather than voltages or currents. This permits an easier definition of reference planes.
I have seen S-parameters described as $S_{11}$, $S_{21}$, etc. Can you explain?

**First lets look at $S_{11}$.**

$S_{11}$ refers to the signal reflected at Port 1 for the signal incident at Port 1.

Scattering parameter $S_{11}$ is the ratio of the two waves $b1/a1$. 
Now let's look at $S_{21}$. $S_{21}$ refers to the signal exiting at Port 2 for the signal incident at Port 1.

$S_{21}$ is correct! $S$-parameter convention always refers to the responding port first!

$S_{21}$? Surely that should be $S_{12}$??
S22 refers to a signal exiting at Port 2 for an incident signal at Port 2. S22 is the ratio of b2/a2.

S12 refers to a signal exiting at Port 1 for an incident signal at Port 2. S12 is the ratio of b1/a2.
I have seen S-parameters described as $S_{11}$, $S_{21}$, etc. Can you explain?

A linear network can be characterised by a set of simultaneous equations describing the exiting waves from each port in terms of incident waves.

$S_{11} = \frac{b_1}{a_1}$

$S_{12} = \frac{b_1}{a_2}$

$S_{21} = \frac{b_2}{a_1}$

$S_{22} = \frac{b_2}{a_2}$

$$ S = \begin{bmatrix} \frac{b_1}{a_1} & \frac{b_2}{a_2} \\ \frac{b_1}{a_2} & \frac{b_2}{a_2} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$ b = Sa $$

Note again how the subscript follows the parameters in the ratio ($S_{11}=b_1/a_1$, etc...)
The transmitted and the reflected wave will have changed in amplitude and phase from the incident wave.

Generally the transmitted and the reflected wave will be at the same frequency as the incident wave.
What do S-parameters depend on?

S-parameters depend upon the network and the characteristic impedances of the source and load used to measure it, and the frequency measured at.

i.e.

if the network is changed, the S-parameters change.

if the frequency is changed, the S-parameters change.

if the load impedance is changed, the S-parameters change.

if the source impedance is changed, the S-parameters change.
This is the matrix algebraic representation of 2 port S-parameters:

\[
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix} =
\begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\times
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
\]

- Some matrices are symmetrical. A symmetrical matrix has symmetry about the leading diagonal.
- In the case of a symmetrical 2-port network, that means that \( S_{21} = S_{12} \) and interchanging the input and output ports does not change the transmission properties.
- A transmission line is an example of a symmetrical 2-port network.
Parameters along the leading diagonal, $S_{11}$ & $S_{22}$, of the S-matrix are referred to as reflection coefficients because they refer to the reflection occurring at one port only.

\[
\begin{pmatrix}
    b_1 \\
    b_2
\end{pmatrix}
= \begin{pmatrix}
    S_{11} & S_{12} \\
    S_{21} & S_{22}
\end{pmatrix}
\times
\begin{pmatrix}
    a_1 \\
    a_2
\end{pmatrix}
\]

Off-diagonal S-parameters, $S_{12}$, $S_{21}$, are referred to as transmission coefficients because they refer to what happens from one port to another.
This is a 4-port network

<table>
<thead>
<tr>
<th>Port 1</th>
<th>Port 2</th>
<th>Port 3</th>
<th>Port 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 ) ( a_2 )</td>
<td>( a_3 ) ( a_4 )</td>
<td>( b_1 ) ( b_2 )</td>
<td>( b_3 ) ( b_4 )</td>
</tr>
</tbody>
</table>

\[
\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \times \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}
\]
Summary

- S-parameters are a powerful way to describe an electrical network.
- S-parameters change with frequency / load impedance / source impedance / network.
- $S_{11}$ is the reflection coefficient.
- $S_{21}$ describes the forward transmission coefficient (responding port 1$^{st}$).
- S-parameters have both magnitude and phase information.
- Sometimes the gain (or loss) is more important than the phase shift and the phase information may be ignored.
- S-parameters may describe large and complex networks.
Outline course

- Resonant cavities
- Rectangular cavity resonators
- Circular cavity resonators
- Q-factor of cavity resonator
- Microwave hybrid circuits
- The S-parameter theory
- Properties of S-parameter
- Three port devices
- Directional couplers
• Power division and combining can be achieved with 3-Port networks.

\[ P_1 = P_2 + P_3 \]

\[ P_2 = \alpha P_1 \]

\[ P_3 = (1-\alpha)P_1 \]

\( \alpha \) is known as the division ratio.

• For a 3-port network, the S-matrix has 9 elements.
• If all ports are matched, then $s_{ii} = 0$, $i=1,2,3...$ (explain this).

$$
S = \begin{bmatrix}
    s_{11} & s_{12} & s_{13} \\
    s_{21} & s_{22} & s_{23} \\
    s_{31} & s_{32} & s_{33}
\end{bmatrix}
$$

• It can be shown that the S-matrix of a 3-port network cannot be **matched**, **lossless** and **reciprocal** at the same time. One of these characteristics has to be given up if the 3-port network is to be physically realizable, see Exercise 1.1 for the mathematical proof.
• A lossless network results in Unitary S-matrix.
• When the S-matrix is non-reciprocal \((s_{ij} \neq s_{ji})\), but the conditions of port match and lossless apply, the 3-port network is known as a Circulator.

\[
S_{cw\_cir} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\]

\[
S_{acw\_cir} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}
\]

Check that both matrices are not symmetry

Note: Here \(\alpha = 1\)

• Circulator usually has ferrite material at the junction to cause the non-reciprocity condition.
• A lossy 3-Port network can be reciprocal and matched at all ports. This type of network is useful as power divider, in addition it can be made to have isolation between its output ports (for instance $s_{23} = s_{32} = 0$

$$S = \begin{bmatrix} 0 & s_{12} & s_{13} \\ s_{12} & 0 & s_{23} \\ s_{13} & s_{23} & 0 \end{bmatrix}$$

• A third type of 3-Port network, which is also used as power divider is reciprocal, lossless and matched at only 2 ports. It can be shown that its S-matrix is given by:

$$S = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$
To fulfill (2) for arbitrary S-parameters, 2 of $s_{12}$, $s_{13}$ and $s_{23}$ must be 0. Substituting this result into (1), it is discovered that (1) cannot be fulfilled. This leads to a contradiction, which shows that our assumption of a 3-port network with matched, reciprocal and lossless conditions is wrong.
The T-junction power divider is a simple 3-port network that can be used for power division or power combining, and can be implemented on stripline, coaxial cable, and waveguide technologies.
Example 1.2: T- Junction Power Divider Design

- A lossless T-junction power divider has a source impedance of $Z_c = 50\Omega$. The impedance is matched at the input. Find the output characteristic impedance so that the input power is divided in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.
- Implement this power divider using microstrip line on a printed circuit board.

\[ P_{in} = \frac{1}{2} \frac{|V|^2}{Z_c} \]

\[ V = V_o^+ + V_o^- = V_o^+ \]

Output powers:

\[ P_1 = \frac{1}{2} \frac{|V|^2}{Z_1} = \frac{1}{3} P_{in} \]

\[ Z_1 = 3Z_c = 150\Omega \]

\[ Z_2 = \frac{3}{2} Z_c = 75\Omega \]

\[ P_2 = \frac{1}{2} \frac{|V|^2}{Z_2} = \frac{2}{3} P_{in} \]

Input impedance to matched divider:

\[ Z_{in} = 75 \parallel 150 = 50\Omega \]

In general this is true for arbitrary power divider ratio, $\alpha$:

\[ Z_m = \frac{\frac{1}{2}Z_c}{\frac{1}{2}Z_c + \frac{1}{2}Z_{in}} = \frac{z_c^2}{(1-\alpha)Z_c + \alpha Z_{in}} = Z_c \]
Looking into the $Z_1 = 150\Omega$ output Tline we see:

$$50 \parallel 75 = 30\Omega \quad \Gamma_1 = \frac{30 - 150}{30 + 150} = -0.666$$

Looking into the $Z_2 = 75\Omega$ output Tline we see:

$$50 \parallel 150 = 37.5\Omega \quad \Gamma_2 = \frac{37.5 - 75}{37.5 + 75} = -0.333$$
Including quarter-wave transformers and step compensations for the T-line...

Quarter wave transformer

Port 1

50Ω

50Ω

150Ω

150Ω

λ/4

50Ω

Port 2

86.60Ω

75Ω

75Ω

61.24Ω

Port 3

Top view

Recommended length of at least quarter wavelength

The observant reader will notice that this realization is a **Narrowband** power divider, by virtue of the wavelength it is only valid at the operating frequency.
Outline course

- Resonant cavities
- Rectangular cavity resonators
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- Three port devices
- **Directional couplers**
• A 4-port network S-matrix contains 16 elements in a 4x4 arrangement.
• Unlike a 3-port network, a 4-port network can be lossless, reciprocal and matched at all ports simultaneously, i.e. the S-matrix has the following form (The matrix is symmetric and unitary):

\[ S = \begin{bmatrix}
0 & s_{12} & s_{13} & s_{14} \\
s_{12} & 0 & s_{23} & s_{24} \\
s_{13} & s_{23} & 0 & s_{34} \\
s_{14} & s_{24} & s_{34} & 0
\end{bmatrix} \]

We have more degree of freedom, so the problem is not over-constrained, unique solution can exist.

• One way for the matrix above to satisfy unitary condition is:

\[ |s_{13}| = |s_{24}|, \quad s_{14} = s_{23} = 0, \quad |s_{12}| = |s_{34}| \]
• It is customary to fix $s_{12}$, $s_{13}$ and $s_{24}$ as:

$$S_{12} = S_{34} = \alpha$$

$$S_{13} = \beta e^{j\delta}$$

$$S_{24} = \beta e^{j\varphi}$$

• Further application of unitary condition yields: $\theta + \phi = \pi \pm 2m\pi$

• Letting $n = 0$, there are 2 choices that is commonly used in practice.

• $\theta = \phi = \pi/2$: $\theta = 0$, $\phi = \pi$:

$$S = \begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0
\end{bmatrix}$$

or

$$S = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0
\end{bmatrix}$$
Directional Coupler (1)

- Consider the matrix of (2.2a), when an incident power wave $a_1$ is directed to port 1 (assuming all ports to be matched):

\[
\begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0 \\
\end{bmatrix}
\begin{bmatrix}
a_1 \\
0 \\
0 \\
0 \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
\alpha a_1 \\
j\beta a_1 \\
0 \\
\end{bmatrix}
\]

- From the theory of S-parameters, the power delivered to port 1, port 2 and port 3 are:

\[P_1 = \frac{1}{2} |a_1|^2 \quad P_2 = \alpha^2 P_1 \quad P_3 = \beta^2 P_1\]

- From the lossless condition of the 4-port network:

\[\alpha^2 + \beta^2 = 1\]
Directional Coupler (2)

- We could repeat the previous exercise, with an incident power wave $a_2$ in port 2.

\[ P_2 = \frac{1}{2}|a_2|^2 \quad P_1 = \alpha^2 P_2 \quad P_4 = \beta^2 P_2 \]

- We conclude that for this particular matched 4-port network, when power is injected into a port, a portion of the power is transmitted to the opposite port while another portion is coupled to the port adjacent to the opposite port. While the adjacent port is isolated.

- This conclusion can also be arrived if we use the matrix of (2.2b), the only difference being the phase of the output voltage waves.
A 4-port network described by S-matrix of (2.2a) or (2.2b) is known as a directional coupler. The following 3 quantities are used to characterize the quality of a directional coupler.

Coupling = $C = 10 \log \frac{R}{P_3} = -20 \log \beta$ dB

Directivity = $D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|}$ dB  \hspace{1cm} (2.3)

Isolation = $I = 10 \log \frac{R}{P_4} = -20 \log |S_{14}|$ dB
Some Typical Directional Couplers

- Hybrid couplers are directional coupler with coupling $C = 3$ dB. This implies
  \[ \alpha = \beta = \frac{1}{\sqrt{2}} \]

- Quadrature hybrid has a $90^\circ$ phase shift between port 2 and 3 when fed at port 1. It is an example of symmetrical coupler.
  \[ S = \frac{1}{\sqrt{2}} \begin{bmatrix}
  0 & 1 & j & 0 \\
  1 & 0 & 0 & j \\
  j & 0 & 0 & 1 \\
  0 & j & 1 & 0
\end{bmatrix} \]

- The magic T or rat-race hybrid has a $180^\circ$ phase difference between port 2 and 3 when fed at port 1, it is an example of anti-symmetrical coupler.
  \[ S = \frac{1}{\sqrt{2}} \begin{bmatrix}
  0 & 1 & 1 & 0 \\
  1 & 0 & 0 & -1 \\
  1 & 0 & 0 & 1 \\
  0 & -1 & 1 & 0
\end{bmatrix} \]
Bi-directional coupler

C = 15 dB

In  fo = 1.83 GHz  Out

Fwd  Rev

FZUP 2002

From: paginas.fe.up.pt/~hmiranda/etele/microstrip/
Outline course

- Circulators and isolators
- Hybrid couplers
**Isolator & Circulator Basics**

An RF **circulator** is a three-port ferromagnetic passive device used to control the direction of signal flow in a circuit and is a very effective, low-cost alternative to expensive cavity duplexers in base station and in-building mesh networks.

An RF **isolator** is a two port ferromagnetic passive device which is used to protect other RF components from excessive signal reflection. **Isolators** are commonplace in laboratory applications to separate a device under test (DUT) from sensitive signal sources.
• Both use the unique properties of ferrites in a magnetic field
• Isolator passes signals in one direction, attenuates in the other
• Circulator passes input from each port to the next around the circle, not to any other port
As described earlier, a common application for a circulator is as an inexpensive duplexer (a device enabling a transmitter and receiver to sharing one antenna). Figure 2 shows that when the transmitter sends a signal, the output goes directly to the antenna port and is isolated from the receiver. Good isolation is key to ensure that a high-power transmitter output signal does not get back to the receiver front end as is governed by the return loss of the antenna. In this configuration, all signals from the antenna go straight to the receiver and not the transmitter because of the circular signal flow (remember the cup of water).

**Clockwise circulation**  
**Counterclockwise circulation**

*Figure 2: Single Isolator*
Figure 3 illustrates the most common application for an isolator. The isolator is placed in the measurement path of a test bench between a signal source and the device under test (DUT) so that any reflections caused by any mismatches will end up at the termination of the isolator and not back into the signal source. This example also clearly illustrates the need to be certain that the termination at the isolated port is sufficient to handle 100% of the reflected power should the DUT be disconnected while the signal source is at full power. If the termination is damaged due to excessive power levels, the reflected signals will be directed back to the receiver because of the circular signal flow.

Figure 3: Single Isolator
When greater isolation is required, a dual junction isolator is used as shown in **Figure 4**. A dual junction isolator is effectively two isolators in series but contained in a single package. Typical isolation performance can range from 40 to 50 dB with this type of design.

**Figure 4: Dual Junction Isolator**
Outline course

- Circulators and isolators
- Hybrid couplers
Hybrids

90° or 180° couplers which split the power equally between direct and coupled ports are called Hybrids

**Quadrature Hybrid (90° phase shift)**

The basic operation of the branch-line coupler is as follows. With all ports matched, power entering port 1 is evenly divided between ports 2 and 3, with a 90° phase shift between these outputs. No power is coupled to port 4 (the isolated port). The scattering matrix has the following form:

\[
[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix}
0 & j & 1 & 0 \\
j & 0 & 0 & 1 \\
1 & 0 & 0 & j \\
0 & 1 & j & 0 \\
\end{bmatrix}
\]
**Quadrature Hybrid (180° phase shift)**

The 180° hybrid junction is a four-port network with a 180° phase shift between the two output ports. It can also be operated so that the outputs are in phase. With reference to the 180° hybrid symbol shown in Figure 7.41, a signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3, and port 4 will be isolated. If the input is applied to port 4, it will be equally split into two components with a 180° phase difference at ports 2 and 3, and port 1 will be isolated. When operated as a combiner, with input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4. Hence, ports 1 and 4 are referred to as the sum and difference ports, respectively. The scattering matrix for the ideal 3 dB 180° hybrid thus has the following form:

\[
[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 0 & -1 \\
1 & 0 & 0 & 1 \\
0 & -1 & 1 & 0 \\
\end{bmatrix}
\]

**FIGURE 7.41** Symbol for a 180° hybrid junction.