Chapter Five

Mechanical Design of Overhead Transmission Lines

Conductor material
The conductor is one of the important items as most of the capital outlay is invested for it. Therefore, proper choice of material and size of the conductor is of considerable importance. The conductor material used for transmission and distribution of electric power should have the following properties:

(i) high electrical conductivity.
(ii) high tensile strength in order to withstand mechanical stresses.
(iii) low cost so that it can be used for long distances.
(iv) low specific gravity so that weight per unit volume is small.

All above requirements are not found in a single material. Therefore, while selecting a conductor material for a particular case, a compromise is made between the cost and the required electrical and mechanical properties.

1-Copper.
Copper is an ideal material for overhead lines owing to its high electrical conductivity and greater tensile strength. It is always used in the hard drawn form as stranded conductor. Although hard drawing decreases the electrical conductivity slightly yet it increases the tensile strength considerably. Copper has high current density i.e., the current carrying capacity of copper per unit of X-sec-tonal area is quite large. This leads to two advantages. Firstly, smaller X-sectional area of conductor is required and secondly, the area offered by the conductor to wind loads is reduced. Moreover, this metal is quite homogeneous, durable and has high scrap value.
There is hardly any doubt that copper is an ideal material for transmission and distribution of electric power. However, due to its higher cost and non-availability, it is rarely used for these purposes. Now-a-days the trend is to use aluminum in place of copper.

2. Aluminum.

Aluminum is cheap and light as compared to copper but it has much smaller conductivity and tensile strength. The relative comparison of the two materials is briefed below:

(i) The conductivity of aluminum is 60% that of copper. The smaller conductivity of aluminum means that for any particular transmission efficiency, the X-sectional area of conductor must be larger in aluminum than in copper. For the same resistance, the diameter of aluminum conductor is about 1.26 times the diameter of copper conductor. The increased X-section of aluminum exposes a greater surface to wind pressure and, therefore, supporting towers must be designed for greater transverse strength. This often requires the use of higher towers with consequence of greater sag.

(ii) The specific gravity of aluminum (2.71 gm/cc) is lower than that of copper (8.9 gm/cc). Therefore, an aluminum conductor has almost one-half the weight of equivalent copper conductor. For this reason, the supporting structures for aluminum need not be made so strong as that of copper conductor.

(iii) Aluminum conductor being light, is liable to greater swings and hence larger cross-arms are required. (iv) Due to lower tensile strength and higher co-efficient of linear expansion of aluminum, the sag is greater in aluminum conductors.

Considering the combined properties of cost, conductivity, tensile strength, weight etc., aluminum has an edge over copper. Therefore, it is being widely used as a conductor material. It is particularly profitable to use aluminum for heavy-current transmission where the conductor size is large and its cost forms a major proportion of the total cost of complete installation.
3. Steel cord aluminum
Due to low tensile strength, aluminum conductors produce greater sag. This prohibits their use for larger spans and makes them unsuitable for long distance transmission. In order to increase the tensile strength, the aluminum conductor is reinforced with a core of galvanized steel wires. The composite conductor thus obtained is known as steel cord aluminum and is abbreviated as A.C.S.R. (aluminum conductor steel reinforced). Steel-cored aluminum conductor consists of central core of † galvanized steel wires surrounded by a number of aluminum strands. Usually, diameter of both steel and aluminum wires is the same. The X-section of the two metals are generally in the ratio of 1 : 6 but can be modified to 1 : 4 in order to get more tensile strength for the conductor. Fig. 8.1 shows steel cord aluminum conductor having one steel wire surrounded by six wires of aluminum. The result of this composite conductor is that steel core takes greater percentage of mechanical strength while aluminum strands carry the bulk of current. The steel cord aluminum conductors have the following advantages:

(i) The reinforcement with steel increases the tensile strength but at the same time keeps the composite conductor light. Therefore, steel cord aluminum conductors will produce smaller sag and hence longer spans can be used.

(ii) Due to smaller sag with steel cord aluminum conductors, towers of smaller heights can be used.

4. Galvanized steel
Steel has very high tensile strength. Therefore, galvanized steel conductors can be used for extremely long spans or for short line sections exposed to abnormally high stresses due to climatic conditions. They have been found very suitable in rural areas where cheap-ness is the main consideration. Due to poor conductivity and high resistance of steel, such conductors are not suitable for transmitting large power over a long distance. However, they can be used to advantage for transmitting a small power over a small distance where the size of the copper conductor
desirable from economic considerations would be too small and thus unsuitable for use because of poor mechanical strength.

5. Cadmium copper.

The conductor material now being employed in certain cases is copper alloyed with cadmium. An addition of 1% or 2% cadmium to copper increases the tensile strength by about 50% and the conductivity is only reduced by 15% below that of pure copper. Therefore, cadmium copper conductor can be useful for exceptionally long spans. However, due to high cost of cadmium, such conductors will be economical only for lines of small X-section i.e., where the cost of conductor material is comparatively small compared with the cost of supports.
Line supports:
The supporting structures for overhead line conductors are various types of poles and towers called line supports. In general, the line supports should have the following properties:
(i) High mechanical strength to withstand the weight of conductors and wind loads etc.
(ii) Light in weight without the loss of mechanical strength.
(iii) Cheap in cost and economical to maintain.
(iv) Longer life.
(v) Easy accessibility of conductors for maintenance.
The line supports used for transmission and distribution of electric power are of various types including wooden poles, steel poles, R.C.C. poles and lattice steel towers.

The choice of supporting structure for a particular case depends upon the line span, X-sectional area, line voltage, cost and local conditions.

1. Wooden poles. These are made of seasoned wood (sal or chir) and are suitable for lines of moderate X-sectional area and of relatively shorter spans, say up to 50 meters. Such supports are cheap, easily available, provide insulating properties and, therefore, are widely used for distribution purposes in rural areas as an economical proposition. The wooden poles generally tend to rot below the ground level, causing foundation failure. In order to prevent this, the portion of the pole below the ground level is impregnated with preservative compounds like creosote oil. Double pole structures of the ‘A’ or ‘H’ type are often used to obtain a higher transverse strength than could be economically provided by means of single poles.

The main objections to wooden supports are:
(i) tendency to rot below the ground level
(ii) comparatively smaller life (20-25 years)
(iii) cannot be used for voltages higher than 20 Kv
(iv) less mechanical strength and
(v) require periodical inspection.
2. Steel poles. The steel poles are often used as a substitute for wooden poles. They possess greater mechanical strength, longer life and permit longer spans to be used. Such poles are generally used for distribution purposes in the cities. This type of supports need to be galvanized or painted in order to prolong its life. The steel poles are of three types viz., (i) rail poles (ii) tubular poles and (iii) rolled steel joints.

3. RCC poles. The reinforced concrete poles have become very popular as line supports in recent years. They have greater mechanical strength, longer life and permit longer spans than steel poles. Moreover, they give good outlook, require little maintenance and have good insulating properties. shows R.C.C. poles for single and double circuit. The holes in the poles facilitate the climbing of poles and at the same time reduce the weight of line supports. The main difficulty with the use of these poles is the high cost of transport owing to their heavy weight. Therefore, such poles are often manufactured at the site in order to avoid heavy cost of transportation.

4. Steel towers. In practice, wooden, steel and reinforced concrete poles are used for distribution purposes at low voltages, say up to 11 kV. However, for long distance transmission at higher voltage, steel towers are invariably employed. Steel towers have greater mechanical strength, longer life, can withstand most severe climatic conditions and permit the use of longer spans. The risk of interrupted service due to broken or punctured insulation is considerably reduced owing to longer spans. Tower footings are usually grounded by driving rods into the earth. This minimizes the lightning troubles as each tower acts as a lightning conductor.

(i) shows a single circuit tower. However, at a moderate additional cost, double circuit tower can be provided (ii). The double circuit has the advantage that it ensures continuity of supply. In case there is breakdown of one circuit, the continuity of supply can be maintained by the other circuit.
SAG AND TENTION CALCULATION:

Let the conductor be suspended between \( P_1 \) and \( P_2 \) with \( o \) as the lowest point in the conductor or curve (original point). Let \( w \) be the weight of the conductor per unit length. Let \( p(x,y) \) be any point on the curve. Draw a tangent at \( p \) as shown in the Fig. above. The tangent making an angle \( \theta \)

1- H: The horizontal tension at \( O \)
2- WS: The weight of wires between \( Op \)
3- Tensions \( T_x \) and \( T_y \) which are the components of tensions in the wire. (\( Ty/Tx = \tan \theta \))

For equilibrium, horizontal and vertical components of the forces must balance so that:
\[ H = T_x \quad \text{and} \quad T_y = WS \]

Also \( \tan \theta = \frac{d_y}{d_x} \)

\[ \frac{d_y}{d_x} = \frac{T_y}{T_x} = \frac{WS}{H} \]

\[
\frac{dy}{dx} = \frac{WS}{H} \quad \text{...}
\]

\[ d_s^2 = d_x^2 + d_y^2 \rightarrow \frac{d_s^2}{d_x^2} = 1 + \frac{d_y^2}{d_x^2} \]

\[ \frac{d_s}{d_x} = \sqrt{1 + \left(\frac{d_y}{d_x}\right)^2} = \sqrt{1 + \frac{w^2 s^2}{H^2}} \]

\[ d_x = \frac{d_s}{\sqrt{1 + \left(\frac{WS}{H}\right)^2}} \], integrating both sides ...

\[ X = \left(\frac{H}{W}\right) \sin h^{-1} \left(\frac{WS}{H}\right) + A \]

Where \( A \) is a constant of integration.

However, referring to the condition at \( x=0 \) where \( s = 0 \), so that \( A = 0 \)

\[ \therefore X = \left(\frac{H}{W}\right) \sin h^{-1} \left(\frac{WS}{H}\right) \]

\[ or \quad \frac{W}{H} \times \frac{W}{H} = \frac{W}{H} \left(\frac{H}{W}\right) \sin h^{-1} \left(\frac{WS}{H}\right) \]

\[ \frac{wx}{H} = \sin h^{-1} \left(\frac{WS}{H}\right) \]

\[ \sin h \left(\frac{WS}{H}\right) = \frac{WS}{H} \]
\[ s = \frac{H}{w} \sin h \left( \frac{w_x}{H} \right) \quad (1) \]

Where \( S \) is the actual length of the conductor op

\[ \frac{dy}{dx} = \frac{w_s}{H} = \sin h \left( \frac{w_x}{H} \right) \]

By integration both sides

\[ y = \frac{H}{w} \cosh \left( \frac{w_x}{H} \right) + B \]

where \( B \) is the constant of integration

When \( y = 0 \rightarrow x = 0 \)

So that \( 0 = \frac{H}{w} + B \rightarrow B = -\frac{H}{w} \)

\[ y = \frac{H}{w} \cosh \left( \frac{w_x}{H} \right) - \frac{H}{w} \]

\[ \therefore y = \frac{H}{w} \left( \cosh \left( \frac{w_x}{H} \right) - 1 \right) \quad (2) \]

Where \( y \) is called the actual say (s) which is a vertical distance between point 0 and the horizontal line between points \( P_1 \) and \( P_2 \) as shown in the Fig.(1)

*The tension \( T \) at point \( P \) is

\[ T^2 = T_x^2 + T_y^2, \quad T_x = H, T_y = ws \]

\[ T^2 = H^2 + W^2 S^2, S^2 = \frac{H^2}{W^2} \sin h^2 \left( \frac{w_x}{H} \right) \]

\[ T^2 = H^2 + W^2 \frac{H^2}{W^2} \sin h^2 \left( \frac{w_x}{H} \right) \]

\[ T^2 = H^2 + H^2 \sin h^2 \left( \frac{w_x}{H} \right) \]
\[ T^2 = H^2 \left( 1 + \sin h^2 \left( \frac{w_x}{H} \right) \right) \] 
\[ T^2 = H^2 \cos h^2 \left( \frac{w_x}{H} \right) \] 
\[ \therefore H \gg w_x \] 
\[ \therefore \cos h \left( \frac{w_x}{H} \right) \times 1 \] 
\[ T = H \cos h \left( \frac{w_x}{H} \right) \]

\[ T \approx H \]  

(3)

*In the eq. (2) of actual sag y or s*

\[ y = \frac{H}{W} \left( \cos h \left( \frac{w_x}{H} \right) - 1 \right) \]

\[ y = \frac{H}{W} \left( 1 + \frac{w_x^2}{2H^2} + \frac{w_x^4}{24} + \ldots + 1 \right) \]

\[ y \approx \frac{w_x^2}{2H} \]

The approximate sag y or s is

\[ y = s \frac{w_x^2}{2T} \]  

(4)

Also \( s = \frac{H}{W} \sin h \left( \frac{w_x}{H} \right) \)

\[ = \frac{H}{W} \left[ \frac{w_x}{H} + \frac{w_x^3}{6H^3} + \ldots + \right] \]

\[ S \approx X + \frac{w_x^2}{6T^2} \]  

(5)
Where $s$ is the approximate length form point P and 0 as an example in the fig.(1)

* In the case of
In the fig.(2)
The distance between
Points $P_1$ and $P_2$ is $\varphi$
*: The sag at point $p$

$$
S = \frac{WX^2}{2T} \cdots \cdots
$$

**The effect of ice**

Let $d$ is the diameter of the conductor
let $t$ is the thickness of the ice
the diameter of the conductor that cover by the ice lager is $d+2t$ as show in the fig.(3)
The volume by $m^3$ that cover one meter
Of the length of the conductor is .:

$$
= \left( \frac{d+2t}{2} \right)^2 \pi - \left( \frac{d}{2} \right)^2 \pi \} \text{ volume } = \text{mass/density}
$$

$$
\frac{\pi}{4}(d + 2t)^2 - \frac{\pi}{4}d^2
$$

Density $= \frac{\text{mass}}{\text{volume}}$

mass $= \text{density } \times \text{volume}$

$$
\text{Weight } = \text{mass}.
$$

*: The weight of one meter
Of the conductor that cover by the ice Lager is
\[ wi = 9400 \times \pi t (d + t) \]

Where 9400 is the density of the ice that cover one meter of the conductor and it is measure by \( N/m^3 \)

\[ \therefore \cdot wt = 9400 \left( \frac{N}{m^3} \right) \times \pi t(d + t)m^2 \]

\[
\begin{align*}
wi &= 2.95 \times 10^4 t(d + t) \\
&\text{N/ one meter}
\end{align*}
\]

Note : The volume of ice lager is equal to the area of ice lager because we take the height of ice lager (cylindrical shape) as one meter.

**The effect of wind ∴**

The effect of the wind will be taken as a horizontal force on the conductor. Assume that the pressure that applied by the wind is \( P \ (N/m^2) \)

\[ \therefore \ w_w = P(d + 2t) \ \\
&\text{N/ one meter} \]

Where \( w_w \) is the force that applied on the conductor by the wind for one meter of the conductor.

**The resultant force**

- \( t \) is the thickness of the ice lager
- \( d \) is the diameter of the conductor
F is the resultant \( p \) (N/m²)
Force by (N) wind pressure

Newton only where

\[
F = \sqrt{\left(\frac{W_w}{1}\right)^2 + \left(W + W_c\right)^2}
\]
N/ one meter

\[
\theta = \tan^{-1}\left(\frac{W_w}{W + W_c}\right)
\]

Where \( W \) is the weight of the one meter of the conductor (N/1 meter)
* Supports at Different Levels

Because of the wind effect, $S_1$ and $S_2$ will not be vertical so that:

$$S_1 = \frac{FX_1^2}{2T} \quad \text{(not vertical)}$$

With the direction of $F$,

$$S_2 = \frac{FX_2^2}{2T} \quad \text{(not vertical)}$$

With the direction of $F$,

$h$ is the difference between the height of the two towers and it’s a vertical, therefore:

$$S_2 \cos \theta - S_1 \cos \theta = h$$

$$\frac{FX_2^2}{2T} \cos \theta - \frac{FX_1^2}{2T} \cos \theta = h$$

$$F \cos \theta = W + W_i = W_t$$
\[ \frac{w_t X_2^2}{2T} - \frac{w_t X_1^2}{2T} = h \]

\( S_1 = \frac{w_t X_1^2}{2T} \) \quad (vertical sag)

\( S_2 = \frac{w_t X_2^2}{2T} \) \quad (vertical sag)

\[ \frac{w_t}{2T} (X_2^2 - X_1^2) = h \]

\[ \frac{w_t}{2T} (X_2 - X_1) (X_2 + X_1) = h \]

\( X_1 + X_2 = l \rightarrow X_2 = l - X_1 \)

\[ \frac{w_t l}{2T} (X_2 - X_1) = h \rightarrow X_2 - X_1 = \frac{2T h}{w_t l} \]

\[ 2 \times 1 = l - \frac{2T h}{w_t l} \rightarrow \begin{align*}
X_1 &= \frac{l}{2} - \frac{T h}{w_t l} \\
X_2 &= l - X_1 \\
X_2 &= l - \left( \frac{l}{2} + \frac{T h}{w_t l} \right) \\
X_2 &= \frac{l}{2} + \frac{T h}{w_t l}
\end{align*} \]

**Safety factor:**

\[ \text{Safety factor} = \frac{\text{maximum Tension}}{\text{actual Tension}} \geq 1 \]

Where the maximum Tension is the tension that design for the conductor and its maximum value. The actual tension is the Tension that the conductor worked at its.

Note:. There is only maximum sag and not found for minimum sag.

\[ \text{Maximum sag} (\delta m) = \frac{F X_m^2}{2T} \]

\( X_m = X_2 \)
Ex1) An overheat transmission line has a radius of conductor of 0.50 m and supported from two towers. When the distance the towers is 150 m, if the two supported point are location on the same level. The conductor is covered by ice lager of the thickness 0.99 cm. the wind pressure is 396 N/m². Find the vertical sag if

- Ice density = 9400 N/m³
- Capper density = 91470 N/m³
- Tensile strength = 17100 N/cm²

So1)

\[ X = \frac{L}{2} \]

The diameter of the conductor

\[ = 2 \times r \]

\[ = 2 \times 0.5 \]

\[ = 1 \text{ cm} \]

The weight of one meter of the conductor.

\[ w_i = 2.95 \times 10^4 t(d + t) \]
\[ w_i = 2.95 \times 10^4 \times 0.94 \times 10^{-2}(1 \times 10^{-2} + 0.94 \times 10^{-2}) \]
\[ w_i = 5.379 \text{ N/one meter} \]

The pressure that effected by the wind on one meter of the conductor is \( w_w \)

\[ w_w = P(d + 2t) \]
\[ w_w = 396 \left(1 \times 10^{-2} + 2 \times 0.94 \times 10^{-2}\right) \]
\[ = 11.4 \text{ N/1 meter} \]
The weight of one meter of the conductor is

\[ W = \left( \frac{a}{2} \right)^2 \pi \times 91470 \quad \text{w = volume \times density} \]

When \( w = \text{weight} = \text{mass} \)

\[ w = \left( \frac{0.01}{2} \right)^2 \pi \times 91470 \]

\[ = 7.18 \text{ N/one meter} \]

The vertical force that effected on the one meter of the conductor

\[ = w_i + w \]

\[ = 5.379 + 7.18 \]

\[ = 12.56 \text{ N/one meter} \]

The resultant force that effect on one Meter of the conductor is \( F \)

\[ F = \sqrt{w + w_i}^2 + w_w^2 \]

\[ F = \sqrt{(12 - 56)^2 + (11.4)^2} \approx 16.97 \approx 17 \text{ N/one meter} \]

The tension on the wires is =

\[ = \text{tensile strength} \times \text{are of the conductor} \]

\[ = 17100 \frac{N}{cm^2} \times (\frac{1}{2})^2 \pi cm^2 = 13430 \text{ N} \]

The sag = \( \delta = \frac{wX^2}{2T} = \frac{F \times X^2}{2T} \quad \text{X} = \frac{\rho}{2} = \frac{150}{2} = 75 \text{ m} \)

The vertical sag = \( \delta \cos \theta = 3.56 \times \frac{w + w_i}{F} = 3.56 \times \frac{12.56}{17} \)

\[ = 2.63 \text{ m} \]
Ex2) An overhead transmission line losses line at a river crossing is supported from two towers a high of 30 and 10 m respectively above water level.

The distance between the towers is 240 m. Find the vertical distance (clearance) between the water level and the conductor in the middle point between the towers.

The tension of the wires is 17800 N and the weight of one meter of the conductor is 742 N.

\[ T = 17800 \text{ N} \]
\[ W = 742 \text{ N/m} \]

\[ h = 60 - 30 = 30 \text{ m} \]

\[ l = 240 \text{ m} \]

\[ X_1 = \frac{l}{2} - \frac{T h}{Wl} \]

\[ X_1 = \frac{240}{2} - \frac{17800 \times 30}{7.42 \times 240} = -180 \text{ m} \]

\[ X_1 = -180 \text{ m} \]

\{ The two towers are in the same side (left or right) due to the original point \}

\[ X_1 = -180 \text{ m} \]

\{ The original point is out of the towers as shown in the Fig. below \}
\[
\delta_1 = \frac{wX_0^2}{2T} = \frac{7.42 \times (180)^2}{2 \times 1.78 \times 1000} = 6.753 \text{ m}
\]

The vertical distance between point 0 and water level 30 - 6.75 = 25 m

\[
\delta_D = \frac{WX_D^2}{2T} \quad \{X_D = X_1 + 120
\]
\[
= 180 + 120
\]
\[
= 300 \text{ M}
\]

\[
\delta_D = \frac{7.42 \times (300)^2}{2 \times 1.78 \times 1000} = 18.75 \text{ m}
\]

The clearance between midpoint between the towers and water level is 18.75 + 23.25 = m

Ex3) An overhead transmission line at a river crossing is supported from two towers. The height of one tower is 52.5 m above water level calculate the high of the other towers in order to make the vertical distance between the conductor and water level 82.5 m in a point has a horizontal distance of 60 m far from the other towers that its height more than 52.5 m. The tension in the wires is 22250 N and the weight of one meter of the conductor is 14.24 N. The distance between the towers is 300 m.
Because the tension in the wires in all point is constant so that we assume the height 82.5 m as a tower. Therefore the new distance $\varphi$ will be between the towers (82.5 and 52.5).

\[
l^- = 300 - 60 = 240 \text{ m} \quad \{ \text{T= 22250 N} \quad \text{W= 14.24 N/1m} \n\]
\[
h^- = 82.5 - 52.5 = 30 \text{ m} \n\]
\[
X_1 = \frac{l^-}{2} - \frac{T \ h^-}{w} \rightarrow \quad X_1 = \frac{240}{2} - \frac{22250 \times 30}{14.24 \times 240} \quad \rightarrow \quad X_1 = -75 \text{ m} \n\]

:. The shape of the towers is correct and the two towers are in the same Side (left or right) due to the original point 0 where point 0 is out of the towers

\[
\delta_1 = \frac{wX_1^2}{2T} = \frac{14.24 \times (75)^2}{2 \times 22250} = 1.8 \text{ m} \n\]

The height from original point and water level $g\ w$ is $g\ w = 52.5 - 1.8 = 50.7 \text{ m}$

\[
X_2 = l + X_1 \rightarrow X_2 = 300 + 75 \rightarrow X_2 = 375 \text{ m} \n\]
\[
\delta_2 = \frac{wX_2^2}{2T} = \frac{14.24 \times (375)^2}{2 \times 22250} = 45 \text{ m} \n\]

The height of the other tower is $45 + 50.7 = 95.7 \text{ m}$.
Ex 3) An overhead transmission line flow through a mount region has a gradient of \( \frac{1}{15} \) and the line is supported from two twoers each of its has a height of 30 m and the distance between the towers is 300m the conductor is cover by ice lager of thickness of 1 cm and has a wind pressure of 380 N/m².

The tension in the wires is 2500 N/cm²

The weight of one meter of the conductor is 9 N.

Also the cross-section area of the conductor is 4 cm² find the height from the original point and ground

Note :. The height in this example means the clearance or the vertical distance.

So1)

\[
A = \left( \frac{d}{2} \right)^2 \pi \rightarrow 4 = \frac{d^2}{4} \pi
\]

\[
d = 2.25 \text{ cm}
\]

\[
w_i = 2.95 \times 10^4 t \ (t + d)
\]

\[
= 2.95 \times 10^4 \times 1 \times 10^{-2} \ (1 \times 10^{-2} + 2.25 \times 10^{-2})
\]

\[
w_i = 9.58 \text{ N/1m}
\]

\[
w_w = P \ (d + 2t)
\]

\[
= 380 \times (2.25 \times 10^{-2} + 2 \times 1 \times 10^{-2})
\]

\[
w_w = 16.15 \text{ N/m}
\]

\[
F = \sqrt{w_w^2 + (w + w_i)^2}
\]

\[
= \sqrt{(16.15)^2 + (9 + 9.58)^2}
\]

\[
F = 24.61 \text{ N/1 m}
\]
\[ T = 2500 \frac{N}{Cm^2} \times 4 \frac{Cm^2}{m^2} = 10,000 \, N \]

\[ X_1 = \frac{\varphi}{2} - \frac{T \, h}{w_t \varphi} \quad \text{ (} w_t = w + w_i \text{)} \]

\[ w_t = 9 + 9.58 = 18.58 \, N/1m \]

\[ \tan \theta = \frac{1}{15} \rightarrow \frac{1}{15} = \frac{FA}{300} \rightarrow FA = 300 \, m \]

\[ h = FA = 20m \]

\[ X_1 = \frac{l}{2} - \frac{T \, h}{w_t \, l} \rightarrow X_1 = \frac{300}{2} - \frac{10,000 \times 20}{10.58 \times 300} \]

\[ X_1 = 114.1135 \, m \]

We see that \( X_1 \) is positive then one tower is on the left of point 0 and the other towers is in the right (i.e the two towers are surrounded) point 0, that mean point 0 location between the two towers.
\[ S_1 = \frac{FX_1^2}{2T} \Rightarrow \delta \frac{w_tX_1^2}{2T} \]

\[ S_1 = \frac{18.58 \times (114.12)^2}{2 \times 10,000} = 12.09 \text{ m} \]

OD = 30 – 12.09 = 17.9 m (vertical distance)
ON = OD – ND

\[ \tan \theta = \frac{NO}{OH} = \frac{ND}{X_1} \Rightarrow \frac{1}{15} = \frac{ND}{114.12} \]

ND = 7.608 m

The vertical distance ON = 17.9 m – 7.608 m = 10.292 m

Ex4) An overhead transmission line at a river crossing is supported from two towers a height of 40 m and 25 m respectively above water level. The distance between the towers is 250 m and the tension in the wires is 9500 N. If the thickness of ice that cover one meter of the conductor is 0.5 cm, and the cross-section area of the conductor is 2 cm². Also the wind pressure is 150 N/m².
Let A any point in the conductor where the vertical distance between this point and the upper point of the long tower is 5 m. find the horizontal distance between point A and the long tower.
Sol)
\[
\begin{align*}
  h &= 40 - 25 = 15 \text{m} \\
  l &= 250 \text{m} \\
  T &= 9500 \text{N} \\
  W &= 3 \text{N} \\
  t &= 0.5 \text{cm} \\
  w &= 150 \frac{N}{m^2}
\end{align*}
\]

\[
2 = \left(\frac{d}{2}\right)^2 \pi = \frac{d^2}{4} \pi
\]

\[
w_i = 2.95 \times 10^4 t(t + d)
\]

\[
= 2.95 \times 10^4 \times 0.5 \times 10^{-2} \times (0.5 \times 10^{-2} + 1.595 \times 10^{-2})
\]

\[
= 3.09 \text{ N/m}
\]

\[
F = \sqrt{w_w^2 + (w + w_i)^2} \quad \rightarrow \quad F = \sqrt{(3.89)^2 + (3 + 3.09)^2}
\]

\[
F = 7.22 \text{ N/m}
\]

\[
w_t = w + w_i \rightarrow w_t 3 + 3.09 = 6.09 \text{ N/m}
\]

\[
X_1 = \frac{l}{2} - \frac{T h}{w_t l} \rightarrow X_1 = \frac{250}{2} - \frac{9500 \times 15}{6.09 \times 250}
\]

\[
X_1 = 31.4 \text{ m}
\]

Because \(X_1\) is positive value

\[
\therefore \text{ point 0 location between the two towers.}
\]
\[ X_2 = l - X_1 \]
\[ = 250 - 31.4 \]
\[ X_2 = 218.6 \text{ m} \]

\[ \delta_2 = \frac{F X_2^2}{2T} \times \cos \theta \quad \text{OR} \quad \delta_2 = \frac{w_t X_2^2}{2T} \]

Where \( w_t = F \cos \theta \)

\[ \delta_2 = \frac{6.09 \times (218.6)^2}{2 \times 9500} = 15.316 \text{ m} \]

\( \delta_2: \text{maximum vertical sag} \)

\[ \delta_A = \delta_2 - 5 \]
\[ = 15.116 - 5 = 10.316 \text{ m} \]

\[ \delta_A = \frac{w_t X_A^2}{2T} \rightarrow 10.316 = \frac{6.09 X_A^2}{2 \times 9500} \rightarrow X_A = 179.4 \text{ m} \]
\[ X = X_2 - X_A \\
= 218.6 - 179.4 \\
= 39.2 \text{ m} \]

OR in other way
Take the towers 35, 25 m respectively

\[ h^- = 35 - 25 = 10 \text{ m} \]

\[ X_1 = \frac{l^-}{2} - \frac{T h^-}{w_t \phi^-} \quad \text{?} \]

\[ 31.4 = \frac{l^-}{2} - \frac{9500 \times 10}{6.09 \times \phi^-} \rightarrow \text{multiply by } l^- \]

\[ 31.4 \phi^- = 0.5 \phi^-^2 - 15599.3 \]

\[ 0.5 \phi^-^2 - 31.4 \phi^- - 15599.3 = 0 \]

\[ l^- = \frac{-(-31.4) \pm \sqrt{(31.4)^2 + 4 \times 0.5 \times 15599.3}}{2 \times 0.5} \]

\[ l^- = 210.8 \text{ m} \]
\[ l^- = -148 \text{ m} \quad \text{neglected} \]

\[ X = l^- - l^- \]

\[ = 250 - 210.8 = 39.19 \text{ m} \]

Also by other towers 40, 35 m respectively

\[ h^- = 40 - 35 = 5 \]

\[ \frac{l^-}{2} - \frac{T h^-}{w_t l^-} \]

\[ X_1 = X_A = -179.4 \text{ m because point 0 became out of the two towers.} \]
\[ X_1 = \frac{l^2}{2} - \frac{T h}{w_t l} \]

\[ -179.4 = \frac{l^2}{2} - \frac{9500 \times 5}{6.09 \times \varphi} \]

\[ -179.4 \varphi = 0.5l^2 - 7799.69 = 0 \]

\[ 0.5l^{-2} + 1799l^{-1} - 7799.67 = 0 \]

\[ l = \frac{-179.4 \pm \sqrt{(179.4)^2 + 4 \times 0.5 \times 7799.6}}{2 \times 0.5} \]

\[ l = 39.19 \text{ m} \]

\[ l = -397.99 \text{ neglected} \]