Chapter Five

Average Permeability

The most difficult reservoir properties to determine usually are the level and distribution of the absolute permeability throughout the reservoir. They are more variable than porosity and more difficult to measure. Yet an adequate knowledge of permeability distribution is critical to the prediction of reservoir depletion by any recovery process. It is rare to encounter a homogeneous reservoir in actual practice. In many cases, the reservoir contains distinct layers, blocks, or concentric rings of varying permeabilities. Also, because smaller-scale heterogeneities always exist, core permeabilities must be averaged to represent the flow characteristics of the entire reservoir or individual reservoir layers (units). The proper way of averaging the permeability data depends on how permeabilities were distributed as the rock was deposited. There are three simple permeability-averaging techniques that are commonly used to determine an appropriate average permeability to represent an equivalent homogeneous system. These are:

• Weighted-average permeability
• Harmonic-average permeability
• Geometric-average permeability

Weighted-Average Permeability

This averaging method is used to determine the average permeability of layered-parallel beds with different permeabilities. Consider the case where the flow system is comprised of three parallel layers that are separated from one another by thin impermeable barriers, i.e., no cross-flow, as shown in Figure 1. All the layers have the same width w with a cross-sectional area of A. The flow from each layer can be calculated by applying Darcy’s equation in a linear form as expressed by equation

$$ q = \frac{kA(p_1 - p_2)}{\mu L} $$

to give:
Figure 1. Linear flow through layered beds.

Layer 1

\[ q_1 = \frac{k_1 w h_1 \Delta p}{\mu L} \]

Layer 2

\[ q_2 = \frac{k_2 w h_2 \Delta p}{\mu L} \]

Layer 3

\[ q_3 = \frac{k_3 w h_3 \Delta p}{\mu L} \]

The total flow rate from the entire system is expressed as

\[ q_t = \frac{k_{avg} w h_t \Delta p}{\mu L} \]
Where $q_t =$ total flow rate

$k_{avg} =$ average permeability for entire model

$w =$ width of the formation

$\Delta p = p_1 - p_2$

$h_t =$ total flow rate

The total flow rate $q_t$ is equal to the sum of the flow rates through each layer or:

$$q_t = q_1 + q_2 + q_3$$

Combining the above expressions gives:

$$\frac{k_{avg} \ w \ h_t \ \Delta p}{\mu \ L} = \frac{k_1 \ w \ h_1 \ \Delta p}{\mu \ L} + \frac{k_2 \ w \ h_2 \ \Delta p}{\mu \ L} + \frac{k_3 \ w \ h_3 \ \Delta p}{\mu \ L}$$

Or

$$k_{avg} \ h_t = k_1 \ h_1 + k_2 \ h_2 + k_3 \ h_3$$

$$k_{avg} = \frac{k_1 \ h_1 + k_2 \ h_2 + k_3 \ h_3}{h_t}$$

The average absolute permeability for a parallel-layered system can be expressed in the following form:

$$k_{avg} = \frac{1}{n} \sum_{j=1}^{n} k_j h_j$$

The above equation is commonly used to determine the average permeability of a reservoir from core analysis data.

Figure 2 shows a similar layered system with variable layers width. Assuming no cross-flow between the layers, the average permeability can be approximated in a manner similar to the above derivation to give:
\[ k_{\text{avg}} = \frac{\sum_{j=1}^{n} k_j A_j}{\sum_{j=1}^{n} A_j} \]

with

\[ A_j = h_j w_j \]

where \( A_j = \) cross-sectional area of layer \( j \)

\( w_j = \) width of layer \( j \)

**Figure 2.** Linear flow through layered beds with variable area.
Radial flow

This case is similar to linear layers except for the circular geometry of the porous medium. All layers have radius $r_e$ and are penetrated by a well of radius $r_w$ as depicted in Fig. 3. The common inlet and outlet pressures are $P_e$ and $P_w$, respectively. For each layer, Darcy's equation is written as:

$$q_i = \frac{2\pi k_i h_i}{\mu} \frac{P_e - P_w}{\ln \frac{r_e}{r_w}}$$

and the total flow rate is

$$q = \frac{2\pi}{\mu} \frac{P_e - P_w}{\ln \frac{r_e}{r_w}} \sum_{i=1}^{i=n} k_i h_i$$

Employing an average permeability, $k$, Darcy's equation for the whole medium will be
Comparing the above equations yields

$$
\bar{k} = \frac{\sum_{i=1}^{i=n} k_i h_i}{\sum_{i=1}^{i=n} h_i} = \frac{\sum_{i=1}^{i=n} k_i h_i}{h}
$$

**Harmonic-Average Permeability**

Permeability variations can occur laterally in a reservoir as well as in the vicinity of a well bore. Consider Figure 4, which shows an illustration of fluid flow through a series combination of beds with different permeabilities.

For a steady-state flow, the flow rate is constant and the total pressure drop $\Delta p$ is equal to the sum of the pressure drops across each bed, or

$$
\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3
$$

Substituting for the pressure drop by applying Darcy’s equation, i.e., Darcy's Equation, gives:

$$
\frac{q \mu L}{A k_{avg}} = \frac{q \mu L_1}{A k_1} + \frac{q \mu L_2}{A k_2} + \frac{q \mu L_3}{A k_3}
$$
Canceling the identical terms and simplifying gives:

\[ k_{\text{avg}} = \frac{L}{(L/k)_1 + (L/k)_2 + (L/k)_3} \]

The above equation can be expressed in a more generalized form to give:

\[ k_{\text{avg}} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} (L/k)_i} \]

where \( L_i \) = length of each bed

\( k_i \) = absolute permeability of each bed

In the radial system shown in Figure 5, the above averaging methodology can be applied to produce the following generalized expression:
The relationship in the above equation can be used as a basis for estimating a number of useful quantities in production work. For example, the effects of mud invasion, acidizing, or well shooting can be estimated from it.

**Geometric-Average Permeability**

Warren and Price (1961) illustrated experimentally that the most probable behavior of a heterogeneous formation approaches that of a uniform system having a permeability that is equal to the geometric average. The geometric average is defined mathematically by the following relationship:

\[
k_{avg} = \exp \left[ \frac{n}{\sum_{j=1}^{n} \frac{\ln \left( \frac{r_j}{r_{j-1}} \right)}{k_j}} \right]
\]

where \( k_i = \) permeability of core sample \( i \)

\( h_i = \) thickness of core sample \( i \)
n = total number of samples If the thicknesses \( h_i \) of all core samples are the same, the above equation can be simplified as follows:

\[
k_{\text{avg}} = (k_1 k_2 k_3 \ldots k_n)^{\frac{1}{n}}
\]

**Absolute Permeability Correlations**

The determination of connate water by capillary-pressure measurements has allowed the evaluation of connate-water values on samples of varying permeability and within a given reservoir to a wider extent and to a greater accuracy than was possible beforehand. These measurements have accumulated to the point where it is possible to correlate connate water content with the permeability of the sample in a given reservoir and to a certain extent between reservoirs.

**The Timur Equation**

Timur (1968) proposed the following expression for estimating the permeability from connate-water saturation and porosity:

\[
k = 8.58102 \frac{\phi^{4.4}}{S_{wc}^2}
\]

**The Morris-Biggs Equation**

Morris and Biggs (1967) presented the following two expressions for estimating the permeability if oil and gas reservoirs:

For a gas reservoir:

\[
k = 2.5 \left( \frac{\phi^3}{S_{wc}} \right)^2
\]

where \( k = \text{absolute permeability, Darcy} \)
\( \phi = \text{porosity, fraction} \)
\( S_{wc} = \text{connate-water saturation, fraction} \).
Examples

Example 1

Given the following permeability data from a core analysis report, calculate the average permeability of the reservoir.

<table>
<thead>
<tr>
<th>Depth, ft</th>
<th>Permeability, md</th>
</tr>
</thead>
<tbody>
<tr>
<td>3998-4002</td>
<td>200</td>
</tr>
<tr>
<td>4002-4004</td>
<td>130</td>
</tr>
<tr>
<td>4004-4006</td>
<td>170</td>
</tr>
<tr>
<td>4006-4008</td>
<td>180</td>
</tr>
<tr>
<td>4008-4010</td>
<td>140</td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>$h_i$, ft</th>
<th>$k_i$</th>
<th>$h_i k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>200</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>260</td>
</tr>
<tr>
<td>2</td>
<td>170</td>
<td>340</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>360</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
<td>280</td>
</tr>
</tbody>
</table>

$h_i = 12$  \[ \sum h_i k_i = 2040 \]

$$k_{avg} = \frac{2040}{12} = 170 \text{ md}$$

Example 2

A hydrocarbon reservoir is characterized by five distinct formation segments that are connected in series. Each segment has the same formation thickness. The length and permeability of each section of the five-bed reservoir are given below:

<table>
<thead>
<tr>
<th>Length, ft</th>
<th>Permeability, md</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
</tbody>
</table>

Calculate the average permeability of the reservoir by assuming:

a. Linear flow system
b. Radial flow system

Solution

For a linear system:

<table>
<thead>
<tr>
<th>$L_i$, ft</th>
<th>$k_i$</th>
<th>$L_i/k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>80</td>
<td>1.8750</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>4.0000</td>
</tr>
<tr>
<td>300</td>
<td>30</td>
<td>10.000</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
<td>25.000</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
<td>20.000</td>
</tr>
<tr>
<td>1350</td>
<td></td>
<td>$\Sigma L_i/k_i = 60.875$</td>
</tr>
</tbody>
</table>

Using equation of average permeability for linear layers ($L_i$) gives:

$$k_{avg} = \frac{1350}{60.875} = 22.18 \text{ md}$$

For a radial system:

The solution of the radial system can be conveniently expressed in the following tabulated form. The solution is based on equation of average permeability for radial layers ($r_i$) and assuming a well bore radius of 0.25 ft:

<table>
<thead>
<tr>
<th>Segment</th>
<th>$r_i$, ft</th>
<th>$\ln(r_i/r_{ab1})$</th>
<th>$k_i$</th>
<th>$[\ln(r_i/r_{ab1})]/k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>well bore</td>
<td>0.25</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>6.397</td>
<td>80</td>
<td>0.080</td>
</tr>
<tr>
<td>2</td>
<td>350</td>
<td>0.847</td>
<td>50</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>650</td>
<td>0.619</td>
<td>30</td>
<td>0.021</td>
</tr>
<tr>
<td>4</td>
<td>1150</td>
<td>0.571</td>
<td>20</td>
<td>0.029</td>
</tr>
<tr>
<td>5</td>
<td>1350</td>
<td>0.160</td>
<td>10</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.163</td>
</tr>
</tbody>
</table>

then,

$$k_{avg} = \frac{\ln (1350/0.25)}{0.163} = 52.72 \text{ md}$$
Example 3
Estimate the absolute permeability of an oil zone with a connate-water saturation and average porosity of 25% and 19%, respectively.

Solution

Applying the Timur equation:

\[
k = 8.58102 \left( \frac{0.19}{0.25} \right)^{4.4} = 0.0921 \text{ Darcy}
\]

From the Morris and Biggs correlation:

\[
k = 62.5 \left[ \frac{0.29}{0.25} \right]^2 = 0.047 \text{ Darcy}
\]

Exercises

1. A linear reservoir \((k = 350 \text{ md}, \varphi = 22\%)\) is 23000 ft long, 3200 ft wide, 65 ft thick and is perfectly horizontal. Oil \((\rho = 55 \text{ lb/ft}^3, \mu = 2.1 \text{ cP})\) enters the reservoir from one end at a pressure of 2940 psi, and it is produced from a well on the other end (see sketch below) at 880 psi pressure. If the well is 12 inches in diameter, compute the steady-state production rate of oil in bbl/d.

Hints:

a. The flow is linear in the first 21400 ft then becomes radial in the last 1600 ft.

b. Only the right-hand half of the well is draining (producing) oil.

2. A horizontal layer of rock has the shape and dimensions shown below.

a. Derive an equation for the steady-state flow of an incompressible fluid through this layer if pressure \(P_1\) is larger than pressure \(P_2\).
3. Derive an equation for the steady state flowrate of an incompressible fluid in a spherical system. In such system, the flow converges from all directions on a hole of radius \( r_w \).

4. A reservoir is 142 feet thick and consists of 7 horizontal zones with different permeabilities as listed below. Compute the average permeability of this reservoir.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Thickness (ft)</th>
<th>Permeability (md)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>280</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>215</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>87</td>
</tr>
<tr>
<td>6</td>
<td>43</td>
<td>300</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>110</td>
</tr>
</tbody>
</table>

5. A well 4” in diameter penetrates a tight gas reservoir (\( h = 76 \text{ ft}, k = 20 \text{ md} \)). A recent test indicated that the reservoir pressure 2000 feet away from the well is 2750 psig. If the well pressure is maintained at 1450 psig, estimate the well production rate in standard cubic feet per day (SCFD). The gas viscosity at the reservoir temperature of 180 °F and various pressures is given below,

<table>
<thead>
<tr>
<th>Pressure (psia)</th>
<th>1450</th>
<th>2100</th>
<th>2750</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity (cP)</td>
<td>0.016</td>
<td>0.019</td>
<td>0.021</td>
</tr>
</tbody>
</table>
6. Because of low permeability, the well of Exercise 5 was acidized. This stimulation process increased the rock permeability to 95 md in a zone only 5 feet in diameter around the well.

a. Estimate the reservoir's average permeability after acidizing.

b. Estimate the well production rate after acidizing

c. How much increase (or decrease) in the well’s production do we gain with acidizing?

7. A 2-mile wide reservoir consists of several layers as shown below. An incompressible liquid with 1.5 cp viscosity flows through the reservoir at steady-state. Compute the flowrate (bbl/day) of the liquid through the layer with k = 200 md.

| P = 750 psia | k = 250 md | h = 50 ft |
| L = 2.4 miles | k = 120 md | h = 30 ft |
| k = 200 md | h = 60 ft |
| k = 70 md | h = 10 ft |
| L = 3.6 miles | P = 2300 psia |