MAYSAN UNIVERSITY
COLLEGE of ENGINEERING
CIVIL ENGINEERING DEPARTMENT
FOURTH YEAR
FOUNDATION DESIGN
COURSE NUMBER CE 402

Syllabus:
- Site Investigation
- Lateral Earth Pressure
- Slope Stability
- Bearing Capacity of Soil
- Settlement
- Foundation Design
- Piles

References

FOUNDATION ANALYSIS AND DESIGN, Fifth Edition, Joseph E. Bowles
PRINCIPLES OF FOUNDATION ENGINEERING, Sixth Edition, Das B M
GEOTECHNICAL ENGINEERING HANDBOOK, Das B M, 2011
1. INTRODUCTION

FOUNDATION DEFINITION AND PURPOSE
All engineered construction resting on the earth must be carried by some kind of interfacing element called a foundation (substructure). The foundation is the part of an engineered system that transmits to, and into, the underlying soil or rock the loads supported by the foundation and its self-weight.

FOUNDATION ENGINEERING
The title foundation engineer is given to that person who by reason of training and experience is sufficiently versed in scientific principles and engineering judgment (often termed "art") to design a foundation. We might say engineering judgment is the creative part of this design process.

The following steps are the minimum required for designing a foundation
1. Locate the site and the position of load.
2. Physically inspect the site for any geological or other evidence that may indicate a potential design problem.
3. Establish the field exploration program.
4. Determine the necessary soil design parameters based on integration of test data, scientific principles, and engineering judgment.
5. Design the foundation using the soil parameters from step 4. The foundation should be economical and be able to be built by the available construction personnel.

GENERAL REQUIREMENTS FOR PROPER DESIGN
1. Determining the building purpose, probable service-life loading, type of framing, soil profile, construction methods, and construction costs
2. Determining the client/owner's needs
3. Making the design, but ensuring that it does not excessively degrade the environment, and provides a margin of safety that produces a tolerable risk level to all parties: the public, the owner, and the engineer.

ADDITIONAL CONSIDERATIONS
1. Depth must be adequate to avoid lateral squeezing of material from beneath the foundation for footings and mats. Similarly, excavation for the foundation must take into account that this can happen to existing building footings on adjacent sites and requires that suitable precautions be taken.
2. Depth of foundation must be below the zone of seasonal volume changes caused by freezing, thawing, and plant growth.
3. The foundation scheme may have to consider expansive soil conditions.
4. In addition to compressive strength considerations, the foundation system must be safe against overturning, sliding, and any uplift (flotation).
5. System must be protected against corrosion or deterioration due to harmful materials present in the soil
6. The foundation should be buildable with available construction personnel.
7. The foundation and site development must meet local environmental standards.
FOUNDATION CLASSIFICATION

Foundations may be classified based on where the load is carried by the ground, producing:

**Shallow foundations**— termed bases, footings, spread footings, or mats. The depth is generally $D/B < 1$ but may be somewhat more.

**Deep foundations**— piles, drilled piers, or drilled caissons. $L_p/B \geq 4$.
<table>
<thead>
<tr>
<th>Foundation Type</th>
<th>Use</th>
<th>Applicable Soil Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shallow Foundations (generally D/B &lt; 1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spread footings, wall footings</td>
<td>Individual columns, walls</td>
<td>Any conditions where bearing capacity is adequate for applied load. May use on a single stratum; firm layer over soft layer or soft layer over firm layer. Check settlements from any source.</td>
</tr>
<tr>
<td>Combined footings</td>
<td>Two to four columns on footing and/or space is limited</td>
<td>Same as for spread footing above</td>
</tr>
<tr>
<td>Mat foundations</td>
<td>Several rows of parallel columns; heavy column loads; use to reduce differential settlements</td>
<td>Soil bearing capacity is generally less than for spread footings, and over half the plan area would be covered by spread footings. Check settlements from any source.</td>
</tr>
<tr>
<td><strong>Deep Foundations (generally L_p / B &gt;= 4')</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floating pile</td>
<td>In groups of 2+ supporting a cap that interfaces with column(s)</td>
<td>Surface and near-surface soils have low bearing capacity and competent soil is at great depth. Sufficient skin resistance can be developed by soil-to-pile perimeter to carry anticipated loads</td>
</tr>
<tr>
<td>Bearing pile</td>
<td>Same as for floating pile</td>
<td>Surface and near-surface soils not relied on for skin resistance; competent soil for point load is at a practical depth (8-20 m).</td>
</tr>
<tr>
<td>Drilled piers or caissons</td>
<td>Same as for piles; use fewer; For large column loads</td>
<td>Same as for piles. May be floating or point-bearing (or combination). Depends on depth to competent bearing stratum</td>
</tr>
<tr>
<td>Retaining wall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retaining walls, bridge abutments</td>
<td>Permanent material retention</td>
<td>Any type of soil but a specified zone, in backfill is usually of controlled fill.</td>
</tr>
<tr>
<td>Sheeting structures (sheet pile, wood sheeting, etc.)</td>
<td>Temporary or permanent for excavations, marine cofferdams for river work</td>
<td>Retain any soil or water. Back for waterfront and cofferdam systems is usually granular for greater drainage</td>
</tr>
</tbody>
</table>
2. SITE INVESTIGATION

It is, actually, the first operation performed in the analysis and design of a foundation. The field and laboratory investigations to obtain the required information are usually termed soil or Site investigation.

For a new structure the soil site investigation should provide data on the following items:

1. Location of the ground water level
2. Bearing capacity of the soil
3. Selection of the alternative type and/or depth of foundation
4. Data on soil parameters and properties so that earth pressure and construction methods may be evaluated.
5. Settlement predictions
6. Potential problems concerning adjacent properties

PLANNING THE PROGRAM

The planning of the subsurface investigation program includes the following:

1. Assembly of all available data and information on the structure
2. Reconnaissance of the area
   - Site visit and physical assessment to the location and the adjacent structures
   - Geological maps
   - Agronomy maps
   - Aerial photograph
   - Water—well logs
   - Hydrological data
   - Highway maps
3. A preliminary site investigation; it is in a form of a few boring or test bits to establish the type and stratification of soil, location of underground water level.
4. A detailed site investigation: for complex projects or where the soil is of poor quality

METHODS of EXPLORATION

1. Drilling holes and open cut

The most widely used method of subsurface investigation for compact sites as well as for most extended sites is boring holes into the ground, from which samples may be collected for either visual inspection or laboratory testing.

2. Field Tests

Standard Penetration Test (SPT)
Cone Penetration Test (CPT)
Vane Shear Test (VST)
Plate Load Test (PLT)

3. Geophysical Methods

To locate boundaries between layers
1. The several exploration methods for sample recovery

<table>
<thead>
<tr>
<th>Disturbed samples taken</th>
<th>Method</th>
<th>Depths</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Auger boring</td>
<td>Depends on equipment and time available, practical depths being up to about 35 m</td>
<td>All soils. Some difficulty may be encountered in gravelly soils. Rock requires special bits, and wash boring is not applicable. Penetration testing is used in conjunction with these methods, and disturbed samples are recovered in the split spoon. Penetration counts are usually taken at 1- to 1.5 m increments of depth.</td>
</tr>
<tr>
<td></td>
<td>Rotary drilling</td>
<td>Depends on equipment, most equipment can drill to depths of 70 m or more</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wash boring</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Percussion drilling</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Test pits and open cuts</td>
<td>As required, usually less than 6 m; use power equipment</td>
<td>All soils</td>
</tr>
<tr>
<td></td>
<td>Undisturbed samples taken</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Auger drilling</td>
<td>Depends on equipment, as for disturbed sample recovery</td>
<td>Thin-walled tube samplers and various piston samplers are used to recover samples from holes advanced by these methods. Commonly, samples of 50- to 100-mm diameter can be recovered</td>
</tr>
<tr>
<td></td>
<td>rotary drilling</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>percussion drilling</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>wash boring</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Test pits</td>
<td>Same as for disturbed samples</td>
<td>Hand-trimmed samples. Careful trimming of sample should yield the least sample disturbance of any method</td>
</tr>
</tbody>
</table>

**NUMBER AND DEPTH of BORING**

There are no clear-cut criteria for determining directly the number and depth of borings required on a project in advance of some subsurface exploration, each site must be carefully considered with engineering judgment in combination with site discovery to finalize the program and to provide an adequate margin of safety.

*For buildings* a minimum of three borings, where the surface is level and the first two borings indicate regular stratification, may be adequate. Five borings are generally preferable (at building corners and center), especially if the site is not level.

*For an antenna or industrial process* tower base in a fixed location, a single boring made at the point may be sufficient.

Additional borings may be required in very uneven sites or where fill areas have been made and the soil varies horizontally rather than vertically. So the borings should be sufficiently spread to allow this without having to make any (or at least no more than a few) additional borings.

Borings should extend below the depth where the stress increase from the foundation load is significant. This value is often taken as 10 percent (or less) of the contact stress $q_0$. For the
square footing the vertical pressure profile shows this depth to be about 2B. Since footing sizes are seldom known in advance of the borings, a general rule of thumb is 2 x the least lateral plan dimensions of the building or 10 m below the lowest building elevation. Where the 2 X width is not practical as, say, for a one-story warehouse or department store, boring depths of 6 to 15 m may be adequate. On the other hand, for important (or high-rise) structures that have small plan dimensions, it is common to extend one or more of the borings to bedrock or to competent (hard) soil regardless of depth. It is axiomatic that at least one of the borings for an important structure terminates into bedrock if there are intermediate strata of soft or compressible materials.

<table>
<thead>
<tr>
<th>Type of Project</th>
<th>Spacing (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multistory building</td>
<td>10 - 30</td>
</tr>
<tr>
<td>One story industrial plant</td>
<td>20 - 60</td>
</tr>
<tr>
<td>Highway</td>
<td>250 – 500</td>
</tr>
<tr>
<td>Residential subdivision</td>
<td>250 – 500</td>
</tr>
<tr>
<td>Dams and Embankments</td>
<td>40 - 80</td>
</tr>
</tbody>
</table>

Project Categories:

<table>
<thead>
<tr>
<th>Small Structures</th>
<th>Small houses, radio mast, pylons</th>
<th>One borehole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compact projects</td>
<td>Buildings, dams, bridges</td>
<td>Four boreholes beep with close spacing</td>
</tr>
<tr>
<td>Extended projects</td>
<td>Motorways, highways, reservoir</td>
<td>More spaced and shallower boreholes</td>
</tr>
</tbody>
</table>

SOIL SAMPLING
The process of obtaining soil of satisfactory amount and size, with minimum disturbance for conducting the required observation and laboratory tests.

SOIL SAMPLER
Auger Sampler: Used only for classification tests
Split Spoon: a barrel consisting of a length of tube, with outside diameter 50 mm and inside diameter of 35 mm, split lengthwise with a cutting shoe from down and screw coupling from top.
Shelby Tube: thin walled seamless steel or brass tube of diameter 50 or 75 mm and length of 600 – 900 mm, used to collect undisturbed samples, the tube should have \( A_r \leq 10\% \), in order to consider the sample as an undisturbed one.

\[
A_r = \frac{(D_o^2 - D_i^2)}{D_i^2} \times 100
\]

\( A_r \) = Area ratio
\( D_o \) = Outside diameter of the tube
\( D_i \) = Inside diameter of the tube
TYPES of SAMPLES
Disturbed Samples: representative samples from the soil layers are stored in sample jars and brought to the laboratory for inspection and classification, the sample should be sufficient to conduct the required tests; used to cover compaction characteristics, index tests for classification, sieve analysis, Atterberge limit tests and natural water content.
Undisturbed Samples: Actually it is nearly impossible to obtain a truly undisturbed soil samples and the term undisturbed means a sample of minimum disturbance Used to cover consolidation and strength characteristics of soil.

THE STANDARD PENETRATION TEST (SPT)
The standard penetration test (limited in clayey soil), developed around 1927, and is currently the most popular and economical means to obtain subsurface information (both on land and offshore). The method has been standardized as ASTM D 1586 since 1958. The number of blows (N) necessary to produce a penetration of 300mm is recorded as the penetration resistance.
The test consists of the following:
1. Driving the standard split-barrel sampler (split spoon) of dimensions (51 x 610 mm) a distance of 460 mm into the soil at the bottom of the boring.
2. Counting the number of blows to drive the sampler the last two 150 mm distances (total = 300 mm) to obtain the N number.
3. Using a 63.5-kg driving mass (or hammer) falling "free" from a height of 760 mm.

CONE PENETRATION TEST (CPT)
The CPT is a simple test that is now widely used in lieu of the SPT—particularly for soft clays, soft silts, and in fine to medium sand deposits. The test is not well adapted to gravel deposits or to stiff/hard cohesive deposits. This test has been standardized by ASTM as D 3441. In outline, the test consists in pushing the standard cone the ground at a rate of 10 to 20 mm/s and recording the resistance.

VANE SHEAR TEST (VST)
The vane shear test VST is a substantially used method to estimate the in situ undrained shear strength of very soft, sensitive, fine-grained soil deposits. The test is performed by inserting the vane into the soil and applying a torque after a short time lapse, on the order of 5 to 10 minutes.

Plate load test (PLT)
A semi direct method to estimate the bearing capacity of a soil in the field is to apply a load to a model footing and measure the amount of load necessary to induce a given amount of settlement, round steel plate from 6 to 30 inches in diameter are available as well as 1 square foot area plate of 1 inch thickness.
Load increment should be one fifth of the estimated bearing capacity of the soil, time intervals of loading should not be less than one hour and should be approximately of the same duration for all the load increments, the test should continue until a total settlement of 25 mm is obtained.
3. LATERAL EARTH PRESSURE

Introduction

Generally, the lateral earth pressure is the lateral pressure acts between the retaining structure and the soil masses being retained, the magnitude of lateral earth pressure at any depth will depend on the type and amount of wall movement, the shear strength of the soil, the unit weight of the soil, and the drainage conditions. Figure below shows a retaining wall of height \( H \) supporting a soil mass whose shear strength can be defined as:

\[
S = c' + \sigma' \tan \varphi'
\]

Where

\( S \) = shear strength \hspace{1cm} \( c' \) = cohesion \hspace{1cm} \( \sigma' \) = effective normal stress and \hspace{1cm} \( \varphi' \) = effective stress angle of friction.

The essential points are:

1. The effective stress represents the average -Stress carried" by the soil solids and is the difference between the total stress and the pore water pressure.
2. The effective stress principle applies only to normal stresses and not to shear stresses.
3. Deformations of soils are due to effective not total stress.
4. Soils, especially silts and fine sands, can be affected by capillary action.
5. Capillary action results in negative pore water pressures and increases the effective stresses.
6. Downward seepage increases the resultant effective stresses; upward seepage decreases the resultant effective stresses.

Unit weight is the weight of a soil per unit volume; we will use the term bulk unit weight, \( \gamma \), to denote unit weight:

\[
\gamma = \frac{W}{V} = \left( \frac{G_s + S e}{1 + e} \right) \gamma_w
\]

\[
S = \frac{V_w}{V_v}
\]

Relative density \((D_r)\) is an index that quantifies the degree of packing between the loosest and densest possible state of coarse-grained soils as determined by experiments:

\[
D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}}
\]

<table>
<thead>
<tr>
<th>( D_r ) %</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-15</td>
<td>Very loose</td>
</tr>
<tr>
<td>15-35</td>
<td>Loose</td>
</tr>
<tr>
<td>35-65</td>
<td>Medium dense</td>
</tr>
<tr>
<td>65-85</td>
<td>Dense</td>
</tr>
<tr>
<td>85-100</td>
<td>Very dense</td>
</tr>
</tbody>
</table>

Effective or buoyant unit weight is the weight of a saturated soil, surrounded by water, per unit volume of soil

\[
\gamma' = \gamma_{\text{sat}} - \gamma_w = \left( \frac{G_s - 1}{1 + e} \right) \gamma_w
\]
Three conditions may arise related to the degree of wall movement:

1. The wall is restrained from moving, as shown in (a). The effective lateral earth pressure for this condition at any depth is referred to as **at-rest earth pressure**.

   \[ \sigma_h' = K_o \sigma_v' \]

   Where \( K_o = 1 - \sin \theta' \) < 1 for normally consolidated soils

   \( K_o \): At rest earth pressure coefficient.

   \[ K_{oc} = K_o \sqrt{OCR} \]

   \( \sigma_v' = \gamma' \times Z \)

2. The wall may tilt away from the soil that is retained (b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The effective lateral pressure for this condition is referred to as **active earth pressure**. (Smooth vertical wall + horizontal backfilling surface)

   \[ \sigma_h = \sigma_v \tan^2 \left(45 - \frac{\theta}{2}\right) - 2c \tan \left(45 - \frac{\theta}{2}\right) \]

   \( K_a = \tan^2 \left(45 - \frac{\theta}{2}\right) = \frac{1 - \sin \theta}{1 + \sin \theta} \)

   \( K_a \): Coefficient of Active earth pressure which is less than one; means that the active lateral earth pressure being acts on the vertical surface is less than the vertical pressure.

   \[ \sigma_h = \sigma_v K_a - 2c \sqrt{K_a} \]

   Tension crack occurs when the horizontal stress become zero

   \[ 0 = \gamma h_c K_a - 2c \sqrt{K_a} \]

   \[ h_c = \frac{2c}{\gamma \sqrt{K_a}} \]

   Where \( h_c = \text{depth of potential tension crack} \)

   The failure surface corresponding to this state of stresses occurs at a plane of an angle of \( \left(45 + \frac{\theta}{2}\right) \) with the horizontal.

3. The wall may be pushed into the soil that is retained (c). With sufficient wall movement, a soil wedge will fail. The effective lateral pressure for this condition is referred to as **passive earth pressure**.

   The failure occurs at a plane of \( \left(45 - \frac{\theta}{2}\right) \) with the horizontal when the state of a principal stresses

   \[ \sigma_h = \sigma_v \tan^2 \left(45 + \frac{\theta}{2}\right) + 2c \tan \left(45 + \frac{\theta}{2}\right) \]

   \[ K_p = \tan^2 \left(45 + \frac{\theta}{2}\right) = \frac{1 + \sin \theta}{1 - \sin \theta} \]
Means that $K_a < K_o < K_p$ and $K_a = \frac{1}{K_p}$
Coulomb Lateral Earth Pressure Theory

One of the earliest methods for estimating earth pressures against walls, credited to C. A. Coulomb (1776), made a number of assumptions as follows:

1. Soil is isotropic and homogeneous and has both internal friction and cohesion.

2. The rupture surface is a plane surface and the backfill surface is planar (it may slope but is not irregularly shaped).

3. The friction resistance is distributed uniformly along the rupture surface and the soil-to-soil friction coefficient \( f = \tan \phi \)

4. The failure wedge is a rigid body undergoing translation.

5. There is wall friction, i.e., as the failure wedge moves with respect to the back face of the wall a friction force develops between soil and wall. This friction angle is usually termed \( \delta \). The value of \( \delta \) ranges from zero, for smooth surface, to (0.4 \( \phi \) to 0.6 \( \phi \)) for rough surfaces.

6. Failure is a plane strain problem—that is, consider a unit interior slice from an infinitely long wall.

Active Lateral Earth Pressure

The weight of the soil wedge ABE is

\[
W = \gamma A = \frac{\gamma H^2}{2 \sin^2 \alpha} \left( \frac{\sin(\alpha + \rho)}{\sin(\rho - \beta)} \right)
\]

Applying the law of sines

\[
P_a = \frac{W}{\sin(\rho - \Phi)} = \frac{W}{\sin(180^\circ - \alpha - \rho + \Phi + \delta)}
\]

Substituting \( W \) in \( P_a \) getting

\[
P_a = \gamma A = \frac{\gamma H^2}{2 \sin^2 \alpha} \left( \frac{\sin(\alpha + \rho)}{\sin(\rho - \beta)} \right)^2 \frac{W \sin(\rho - \Phi)}{\sin(180^\circ - \alpha - \rho + \Phi + \delta)}
\]

The value of \( P_a \) depends only on \( \rho \), since all other items are constant for a given problem, and the maximum value of \( P_a \) can be obtained by setting \( \frac{dP_a}{d\rho} = 0 \).
\[ P_a = \frac{\gamma H^2}{2} \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta) \left[1 + \frac{\sin(\phi + \delta) \sin(\phi - \beta)}{\sin(\alpha - \delta) \sin(\alpha + \beta)}\right]^2} \]

If \( \beta = 0 \) horizontal backfilling, \( \delta = 0 \) smooth wall and \( \alpha = 90^\circ \) vertical wall, then
\[ P_a = \frac{\gamma H^2}{2} \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{\gamma H^2}{2} \tan^2\left(45^\circ - \frac{\phi}{2}\right) \]
\[ P_a = \frac{\gamma H^2}{2} K_a \]

In general
\[ K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta) \left[1 + \frac{\sin(\phi + \delta) \sin(\phi - \beta)}{\sin(\alpha - \delta) \sin(\alpha + \beta)}\right]^2} \]

**Passive Lateral Earth Pressure**

Similarly, the passive lateral earth pressure can be found using the same principles used in finding the active lateral earth pressure.

The wedge of the failed soil is
\[ W = \gamma A = \frac{\gamma H^2}{2} \left(\sin(\alpha + \rho) \frac{\sin(\alpha + \beta)}{\sin(\rho - \beta)}\right) \]

Using law of sines passive force is
\[ P_p = \frac{W \sin(\rho + \phi)}{\sin(180^\circ - \rho - \phi - \delta - \alpha)} \]

Substituting weight of soil in the above equation of the passive force, then
\[ \frac{dP_p}{d\rho} = 0 \rightarrow P_p = \frac{\gamma H^2}{2} \frac{\sin^2(\alpha - \phi)}{\sin^2\alpha \sin(\alpha + \delta) \left[1 - \frac{\sin(\phi + \delta) \sin(\phi + \beta)}{\sin(\alpha + \delta) \sin(\alpha + \beta)}\right]^2} \]

For vertical smooth wall with horizontal backfilling surface \( (\alpha = 90^\circ, \delta = \beta = 0) \)
\[ P_p = \frac{\gamma H^2}{2} \left( 1 + \frac{\sin \varphi}{1 - \sin \varphi} \right) = \frac{\gamma H^2}{2} \tan^2 \left( 45 + \frac{\varphi}{2} \right) \]

In general

\[ K_p = \frac{\sin^2(\alpha - \varphi)}{\sin^2\alpha \sin(\alpha + \delta) \left( 1 - \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi + \beta)}{\sin(\alpha + \delta) \sin(\alpha + \beta)}} \right)^2} \]

**Rankine Earth Pressure Theory**

Rankine (1857) considered soil in a state of plastic equilibrium and used essentially the same assumptions as Coulomb, except that he assumed no wall friction or soil cohesion. The Rankine case gives the pressure ratio acting parallel to backfill

\[ K_\alpha = \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \varphi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \varphi}} \]

\[ K_p = \cos \beta \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \varphi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \varphi}} \]

---

(a) Soil-structure system for the Rankine solution for \( \alpha = 90^\circ \)

(b) Force triangle in the Rankine solution
Example: Determine the total active force/unit width of wall and its location? Use the Coulomb equations and take a smooth wall.

Solution:

Critical Thinking

Use coulomb equations for $K_a$ for the wall and both soils with $\alpha=90^\circ$ for vertical wall $\beta=0^\circ$ for Level backfill surface $\delta=0^\circ$ for smooth wall

For saturated soil (soil 2) use buoyant unit weight to find lateral pressure

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$K_{a1} = 0.307$  $K_{a2} = 0.333$

At top $h=0$

$\sigma_a = 100 \times 0.307 = 30.7$ kPa

At $h=3.5-dh$

$\sigma_a = 30.7 + 16.5 \times 3.5 \times 0.307 = 30.7 + 17.73 = 48.43$ kPa

At $h=3.5+dh$

$\sigma_a = 100 \times 0.333 + 16.5 \times 3.5 \times 0.333 = 33.3 + 52.55 = 85.85$ kPa

At $h=7$

$\sigma_a = 33.3 + (16.5 + (19.25-9.81)) \times 3.5 \times 0.333 = 33.3 + 11 = 63.5$ kPa

At $h=7$ due water

$\sigma_a = 3.5 \times 9.81 = 34.33$ kPa
Example: Determine the total active force per unit width of wall for the soil-wall system, using the Coulomb equations? Where does $P_a$ act?

Solution:

**Critical Thinking**

Use Coulomb equations of $K_a$ for the wall and soil with $\alpha=90^\circ$ for vertical wall $\beta=10^\circ$ for inclined backfill surface $\delta=2\phi/3$ (practical estimate) for rough wall $\gamma=20^\circ$

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta) \left(1 + \frac{\sin(\phi + \delta) \sin(\phi - \beta)}{\sin(\alpha - \delta) \sin(\alpha + \beta)}\right)^2}$$

$$K_a = \frac{\sin^2(90 + 30)}{\sin^2 90 \sin(90 - 20) \left(1 + \frac{\sin(30 + 20) \sin(30 - 10)}{\sin(90 - 20) \sin(90 + 10)}\right)^2} = 0.34$$

At base $P_a = 0.5 \times 17.52 \times 5^2 \times 0.34 = 74.46 \text{ kN}$. Acts on 5/3 from base by angle of 20 degree with the normal to the wall.

If we use Rankine theory

$$K_a = \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

$$K_a = \cos 10 \frac{\cos 10 - \sqrt{\cos^2 10 - \cos^2 30}}{\cos 10 + \sqrt{\cos^2 10 - \cos^2 30}} = 0.3495$$

At base $P_a = 0.5 \times 17.52 \times 5^2 \times 0.349 = 76.54 \text{ kN}$. Acts on 5/3 from base by angle of 10 degrees (same angle of inclined surface) with the normal to the wall.

---

2015 - 2016
Example: Determine the active earth pressure for the wall soil profile shown. Using the Rankine equations?

Solution:

Critical Thinking

Use Rankine equations of $K_a$ for the wall and soil with

- $\alpha = 90$ for vertical wall
- $\beta = 0$ for level backfill surface
- $\delta = 0$ (same as $\beta$)

For saturated soils (2, 3, 4, 5) Use buoyant unit weight to find Lateral pressure

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$\sigma_h = \sigma_v K_a - 2c\sqrt{K_a}$$

<table>
<thead>
<tr>
<th>soil</th>
<th>Depth m</th>
<th>$\phi$</th>
<th>$K_a$</th>
<th>$\gamma'$ kN/m$^3$</th>
<th>$\sigma_v$ kPa</th>
<th>$\sigma_h$ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>32</td>
<td>.307</td>
<td>17.30</td>
<td>100</td>
<td>30.7</td>
</tr>
<tr>
<td></td>
<td>1.8-dz</td>
<td></td>
<td></td>
<td></td>
<td>100+17.3x1.8= 131.14</td>
<td>40.3</td>
</tr>
<tr>
<td>2</td>
<td>1.8+dz</td>
<td>0</td>
<td>1</td>
<td>9.79</td>
<td>131.14</td>
<td>-8.9</td>
</tr>
<tr>
<td></td>
<td>2.4-dz</td>
<td></td>
<td></td>
<td></td>
<td>131.14+9.79x0.6=137</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>2.4+dz</td>
<td>10</td>
<td>.704</td>
<td>9.89</td>
<td>137</td>
<td>44.8</td>
</tr>
<tr>
<td></td>
<td>5.15-dz</td>
<td></td>
<td></td>
<td></td>
<td>137+9.89x2.75=164.22</td>
<td>65.3</td>
</tr>
<tr>
<td>4</td>
<td>5.15+dz</td>
<td>0</td>
<td>1</td>
<td>9.19</td>
<td>164.22</td>
<td>84.2</td>
</tr>
<tr>
<td></td>
<td>7.6-dz</td>
<td></td>
<td></td>
<td></td>
<td>164.22+9.19x2.45=186.7</td>
<td>106.7</td>
</tr>
<tr>
<td>5</td>
<td>7.6+dz</td>
<td>20</td>
<td>.49</td>
<td>9.19</td>
<td>186.7</td>
<td>63.5</td>
</tr>
<tr>
<td></td>
<td>9.1</td>
<td></td>
<td></td>
<td></td>
<td>186.7+1.5x9.19=199</td>
<td>69.5</td>
</tr>
</tbody>
</table>

At base $\sigma_{hw}=9.81x 7.3= 71.61$ kPa due water
Example: Draw the active earth pressure diagram for a unit width of wall for the conditions shown?

Solution:

Critical Thinking

Use Rankine equations of $K_a$ for the wall and soil with

$\alpha=90^\circ$ for vertical wall \hspace{1mm} $\beta=0^\circ$ for level backfill surface \hspace{1mm} $\delta=0^\circ$ (same as $\beta$)

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\sigma_h = \sigma_v K_a - 2c \sqrt{K_a}$$

To find the net horizontal pressure on the wall, two scenarios can be followed

1. Neglect the tensile stresses
2. Start from zero stress at top

$K_a = \frac{1 - \sin 10^\circ}{1 + \sin 10^\circ} \approx 0.704$

At top

$$\sigma_h = -2 \times 10.5 \times \sqrt{0.704} = -17.62 \text{ kPa}$$

$\sigma_h = 0$ at:

$$0 = 17.52 \times h \times 0.704 - 2 \times 10.5 \times \sqrt{0.704} \hspace{1mm} h = 1.43 \text{ m}$$

At base

$$\sigma_h = 17.52 \times 6.5 \times 0.704 - 2 \times 10.5 \times \sqrt{0.704} = 62.53 \text{ kPa}$$

If water fills the cracks at top it develops active pressure on wall of

$$\sigma_{hw} = 9.81 \times 1.43 \times 1 = 14 \text{ kPa}$$
4-BEARING CAPACITY OF SOILS

INTRODUCTION
The soil must be capable of carrying the loads from any engineered structure placed upon it without a shear failure and with the resulting settlements being tolerable for that structure. The evaluation of the limiting shear resistance or ultimate bearing capacity \( q_{ult} \) of the soil under a foundation load, depends on the soil strength perimeters. The recommendation for the allowable bearing capacity \( q_a \) to be used for design is based on the \textit{minimum} of either

1. Limiting the settlement to a tolerable amount.
2. The ultimate bearing capacity, which considers soil strength, where \( q_a = \frac{q_{ult}}{F} \). Where \( F \) is the factor of safety, which typically is taken as 3 for the bearing capacity of shallow foundations.

Probably, one of two potential modes of failure may occur when a footing loaded to produce the ultimate bearing capacity of the soil.

a. Rotate about some center of rotation (probably along the vertical line oa) with shear resistance developed along the perimeter of the slip zone shown as a circle
b. Punch into the ground as the wedge agb.

\[ \text{a. Footing on } \phi = 0 \text{ Soil} \]

\[ \text{Upper Bound Solution} \]
\[ \sum m_o = 0 \Rightarrow \]
\[ q B \times \frac{B}{2} + c \pi B \times B - q_{ult} B \times \frac{B}{2} = 0 \]
\[ \text{yields} \]
\[ q_{ult} = 2c\pi + \overline{q} \]

\[ \text{Lower Bound Solution} \]
At point o, left side element 1 and adjacent right side element 2, the horizontal stresses should be equal \( \sigma_{3,1} = \sigma_{1,2} \)

\[ q_{ult} \tan^2 \left(45 - \frac{\phi}{2}\right) - 2c \tan \left(45 - \frac{\phi}{2}\right) = \bar{q} \tan^2 \left(45 + \frac{\phi}{2}\right) + 2c \tan \left(45 + \frac{\phi}{2}\right) \]

For \( \phi = 0 \), \( \tan 45° = 1 \) then

\[ q_{ult} = 4c + \bar{q} \]

For the case of \( \bar{q} = 0 \), i.e. foundation on ground surface, the average value is \( q_{ult} = 5.14c \)

The safe self-standing depth of excavation (Z) occurs when \( q_{ult} = 0 = 4c + q' \) gives ;

\[ Z = \frac{4c}{\gamma} \]
b. Footing on $\phi$-c Soil

\[
\alpha = 45 + \phi/2
\]
\[
H = \frac{B}{2} \tan \alpha
\]
\[
A = \frac{B}{2 \cos \alpha}
\]
\[
P_{p.v} = \frac{P_p}{\cos \phi}
\]

\[
W = \frac{1}{2} \gamma B D \tan \alpha
\]
\[
K_s = \tan^2 \left(45 + \frac{\phi}{2}\right)
\]
\[
K_v = \tan^2 \left(45 - \frac{\phi}{2}\right)
\]

---

Neglect shear
(Terzaghi, Hansen)

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2015 - 2016
BEARING CAPACITY EQUATIONS

Terzaghi Bearing-Capacity Equation

One of the early sets of bearing-capacity equations was proposed by Terzaghi (1943) for Shallow (D ≤ B) strip footing. Terzaghi’s equations were produced from a slightly modified bearing-capacity theory developed by Prandtl (1920) but Terzaghi used shape factors.

\[ q_{ult} = c N_c s_c + \bar{q} N_q + 0.5 B Y N_y s_y \]

Where

C= cohesion of soil
\( \phi \)= angle of internal friction of soil
B= least lateral dimension of footing (diameter for round footing).
\( \gamma \)= unit weight of soil (use buoyant (submerged) unit weight for soil below water table.
\( \bar{q} \)= \( \gamma \)D

\[ N_q = \frac{a^2}{\cos^2(45 + \phi/2)} \]

\[ N_c = (N_q - 1) \cot \phi \]

\[ N_y = \frac{\tan \phi}{2} (\frac{K_{py}}{\cos^2 \phi} - 1) \]

Conclusions

1. The ultimate bearing capacity increases with the depth of footing.
2. The ultimate baring capacity of cohesive soil (\( \phi = 0 \)) is independent of footing size; i.e., at ground level \( q_{ult} = 5.7c \)
3. The ultimate bearing capacity of a cohesionless soil (c=0) is directly dependent on the footing size, but the depth of footing is more important than size.

Meyerhof’s Bearing-Capacity Equation

Meyerhof (1951, 1963) proposed a bearing-capacity equation similar to that of Terzaghi but included a shape factor \( S_q \) with the depth term \( N_q \). He also included depth factors \( d_b \) and inclination factors \( i_i \) for cases where the footing load is inclined from the vertical. Up to a depth of D \( \approx \) B the Meyerhof \( q_{ult} \) is not greatly different from the Terzaghi value. The difference becomes more pronounced at larger D/B ratios.

\[ q_{ult} = c N_c s_c d_c + \bar{q} N_q s_q d_q + 0.5 B Y N_y s_y d_y \]

Vertical load

\[ q_{ult} = c N_c d_c i_c + \bar{q} N_q d_q i_q + 0.5 B Y N_y d_y i_y \]

Inclined load

Meyerhof’s Bearing-Capacity Table

\[ \begin{array}{c|c|c|c|c} \hline \phi \degree & N_c & N_q & N_y \\
\hline 0 & 5.7 & 1.0 & 0.0 \\
5 & 7.3 & 1.6 & 0.5 \\
10 & 9.6 & 2.7 & 1.2 \\
15 & 12.9 & 4.4 & 2.5 \\
20 & 17.7 & 7.4 & 5.0 \\
25 & 25.1 & 12.7 & 9.7 \\
30 & 37.2 & 22.5 & 19.7 \\
35 & 52.6 & 36.5 & 36.0 \\
40 & 57.8 & 41.4 & 42.4 \\
45 & 95.7 & 81.3 & 100.4 \\
48 & 172.3 & 173.3 & 297.4 \\
48 & 258.3 & 287.9 & 780.1 \\
50 & 347.5 & 415.1 & 1153.2 \\
\hline \end{array} \]

\[ \begin{array}{c|c|c|c|c|c|c} \hline \text{Shape Factor} & \text{Strip} & \text{Round} & \text{Square} & \text{Rectangle} \\
\hline s_c & 1 & 1.3 & 1.3 & 1+ 0.3B/L \\
\hline s_y & 1 & 0.6 & 0.8 & 1- 0.2B/L \\
\hline \end{array} \]
\[ N_q = e^{(\pi \tan \phi)} \tan^2 \left( 45 + \frac{\phi}{2} \right) \]
\[ N_c = (N_q - 1) \cot \phi \]
\[ N_y = (N_q - 1) \tan(1.4 \phi) \]

Shape, depth and inclination factors

**Hansen’s Bearing-Capacity Method**

Hansen (1970) proposed the general bearing-capacity case and N factor equations. This equation is readily seen to be a further extension of the earlier Meyerhof work. The extensions include base factors for situations in which the footing is tilted from the horizontal \( b_i \) and for the possibility of a slope \( \beta \) of the ground supporting the footing to give ground factors \( g_i \). The Hansen equation implicitly allows any D/B and thus can be used for both shallow and deep foundation.

\[ q_{ult} = c N_c s_c d_c i_c g_c b_c + q N_q s_q d_q i_q g_q b_q + + 0.5 B \gamma N_y s_y d_y i_y g_y b_y \]

When \( \phi = 0 \)

\[ q_{ult} = 5.14 S_u (1 + s'_c + d'_c - i'_c - b_c - g_c) + \bar{q} \]

\[ N_q = e^{(\pi \tan \phi)} \tan^2 \left( 45 + \frac{\phi}{2} \right) \]
\[ N_c = (N_q - 1) \cot \phi \]
\[ N_y = 1.5 (N_q - 1) \tan(\varnothing) \]

\( s_u \) = undrained \( \phi = 0 \) ultimate shear strength

**Vesic’s Bearing-Capacity Equations**

The Vesic (1973, 1975) procedure is essentially the same as the method of Hansen (1961) with select changes. The \( N_c \) and \( N_q \) terms are those of Hansen but \( N_y \) is slightly different. There are also differences in the \( i_b, b_i, g_i \) terms. The Vesic equation is somewhat easier to use than Hansen’s because Hansen uses the / terms in computing shape factors \( s_i \) whereas Vesic does not.

\[ q_{ult} = c N_c s_c d_c i_c g_c b_c + q N_q s_q d_q i_q g_q b_q + + 0.5 B \gamma N_y s_y d_y i_y g_y b_y \]

\[ N_q = e^{(\pi \tan \phi)} \tan^2 \left( 45 + \frac{\phi}{2} \right) \]
\[ N_c = (N_q - 1) \cot \phi \]
\[ N_y = (2N_q + 1) \tan(\varnothing) \]
Shape and depth factors for use in either the Hansen (1970) or Vesič (1973, 1975b) bearing-capacity equations of Table 4-1. Use $\gamma_d$, $d_r$ when $\phi = 0$ only for Hansen equations. Subscripts $H$, $V$ for Hansen, Vesič, respectively.

<table>
<thead>
<tr>
<th>Shape factors</th>
<th>Depth factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{phi} = 0.5 \frac{B}{L}$ ($\phi = 0$)</td>
<td>$d_r = 0.4k$ ($\phi = 0$)</td>
</tr>
<tr>
<td>$x_{phi} = 1.0 + \frac{N_d}{N_r}$</td>
<td>$k = \frac{D/B}{D/B' \leq 1}$</td>
</tr>
<tr>
<td>$x_{phi} = 1.0 + \frac{N_d}{N_r}$</td>
<td>$k = \tan^{-1}(D/B')$ for $D/B &gt; 1$</td>
</tr>
<tr>
<td>$x_{phi} = 1.0$ for strip</td>
<td>$k$ in radians</td>
</tr>
<tr>
<td>$x_{phi} = 1.0 - \frac{B}{L}$ $\sin \phi$</td>
<td>$d_r = 1 + 2\tan \phi (1 - \sin \phi)^{1/2}k$</td>
</tr>
<tr>
<td>$x_{phi} = 1.0 - \frac{B}{L} \tan \phi$</td>
<td>$k$ defined above</td>
</tr>
</tbody>
</table>

for all $\phi$

Notes:
1. Note use of "effective" base dimensions $B'$, $L'$ by Hansen but not by Vesič.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load $H_d$.
3. With a vertical load and a load $H_d$, and either $H_d - 0$ or $H_d > 0$ you may have to compute two sets of shape $\gamma_d$ and $d_r$, on $\gamma_d$, $i_d$, $i_d$, and $d_r$, $d_r$, respectively. For the subscripts of $H_d$, $V_d$, $N_d$, $N_r$, presented in Sec. 4-6, use ratios $L/B'$ or $D/B'$.

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5c for equivalent Vesič equations.

<table>
<thead>
<tr>
<th>Inclination factors</th>
<th>Ground factors (base on slope)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1 = 0.5 - \frac{1}{V + \gamma_1 C_1}$</td>
<td>$s_r = \frac{\beta^2}{147}$</td>
</tr>
<tr>
<td>$i_2 = \frac{1 - i_1}{N_d - 1}$</td>
<td>$s_r = \frac{1.0 - \beta^2}{147}$</td>
</tr>
<tr>
<td>$i_3 = \sqrt{1 - \frac{0.5H_d}{V + \gamma_d C_d \cos \phi}}$</td>
<td>$s_r = s_r = (1 - 0.5\tan \beta)^2$</td>
</tr>
<tr>
<td>$s_r \leq 5$</td>
<td></td>
</tr>
</tbody>
</table>

Base factors (filled base)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1 = 1 + \frac{0.7H_d}{V + \gamma_1 C_1}$</td>
<td>$b_r = \frac{\gamma_2}{147}$ ($\phi = 0$)</td>
</tr>
<tr>
<td>$i_2 = 1 - \frac{0.7H_d}{V + \gamma_2 C_2 \cos \phi}$</td>
<td>$b_r = 1 - \frac{\gamma_2}{147}$ ($\phi = 0$)</td>
</tr>
<tr>
<td>$i_3 = \frac{1 + 0.7(2\gamma_2)\cos \phi}{V + \gamma_2 C_2 \cos \phi}$</td>
<td>$b_r = \exp \left[-0.1\tan \phi \right]$</td>
</tr>
<tr>
<td>$s_r \leq 5$</td>
<td>$b_r = \exp \left[-0.1\tan \phi \right]$</td>
</tr>
</tbody>
</table>

Note:
1. Use $H_d$ as either $H_d$ or $H_d$ or both if $H_d > 0$.
2. Hansen (1970) did not give an $i_1$ for $\phi = 0$. The value above is from Hansen (1961) and also used by Vesič.
3. Variable $C_d$ is base adhesion, on the order of 0.0 to 1.0 x base cohesion.
4. Refer to sketch for identification of angles $i$ and $\beta$, footing depth $D$, location of $H_d$, $N_d$, and top of base slab (usually also produces eccentricity). Especially note: $V$ = force normal to base and is not the resultant $R$ from combining $V$ and $H_d$. 

Bearing-capacity factors for the Meyerhof, Hansen, and Vesič bearing-capacity equations

Note that $N_c$ and $N_q$ are the same for all three methods; subscripts identify author for $N_q$.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$N_c$</th>
<th>$N_q$</th>
<th>$N_{y(H)}$</th>
<th>$N_{y(M)}$</th>
<th>$N_{y(V)}$</th>
<th>$N_{y}/N_c$</th>
<th>$2\tan \phi (1 - \sin \phi)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.14*</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.195</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>6.49</td>
<td>1.6</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td>0.242</td>
<td>0.146</td>
</tr>
<tr>
<td>10</td>
<td>8.34</td>
<td>2.5</td>
<td>0.4</td>
<td>0.4</td>
<td>1.2</td>
<td>0.296</td>
<td>0.241</td>
</tr>
<tr>
<td>15</td>
<td>10.97</td>
<td>3.9</td>
<td>1.2</td>
<td>1.1</td>
<td>2.6</td>
<td>0.359</td>
<td>0.294</td>
</tr>
<tr>
<td>20</td>
<td>14.83</td>
<td>6.4</td>
<td>2.9</td>
<td>2.9</td>
<td>5.4</td>
<td>0.431</td>
<td>0.315</td>
</tr>
<tr>
<td>25</td>
<td>20.71</td>
<td>10.7</td>
<td>6.8</td>
<td>6.8</td>
<td>10.9</td>
<td>0.514</td>
<td>0.311</td>
</tr>
<tr>
<td>26</td>
<td>22.25</td>
<td>11.8</td>
<td>7.9</td>
<td>8.0</td>
<td>12.5</td>
<td>0.533</td>
<td>0.308</td>
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<tr>
<td>28</td>
<td>25.79</td>
<td>14.7</td>
<td>10.9</td>
<td>11.2</td>
<td>16.7</td>
<td>0.570</td>
<td>0.299</td>
</tr>
<tr>
<td>30</td>
<td>30.13</td>
<td>18.4</td>
<td>15.1</td>
<td>15.7</td>
<td>22.4</td>
<td>0.610</td>
<td>0.289</td>
</tr>
<tr>
<td>32</td>
<td>35.47</td>
<td>23.2</td>
<td>20.8</td>
<td>22.0</td>
<td>30.2</td>
<td>0.654</td>
<td>0.276</td>
</tr>
<tr>
<td>34</td>
<td>42.14</td>
<td>29.4</td>
<td>28.7</td>
<td>31.1</td>
<td>41.0</td>
<td>0.698</td>
<td>0.262</td>
</tr>
<tr>
<td>36</td>
<td>50.55</td>
<td>37.7</td>
<td>40.0</td>
<td>44.4</td>
<td>56.2</td>
<td>0.746</td>
<td>0.247</td>
</tr>
<tr>
<td>38</td>
<td>61.31</td>
<td>48.9</td>
<td>56.1</td>
<td>64.0</td>
<td>77.9</td>
<td>0.797</td>
<td>0.231</td>
</tr>
<tr>
<td>40</td>
<td>75.25</td>
<td>64.1</td>
<td>79.4</td>
<td>93.6</td>
<td>109.3</td>
<td>0.852</td>
<td>0.214</td>
</tr>
<tr>
<td>45</td>
<td>133.73</td>
<td>134.7</td>
<td>200.5</td>
<td>262.3</td>
<td>271.3</td>
<td>1.007</td>
<td>0.172</td>
</tr>
<tr>
<td>50</td>
<td>266.50</td>
<td>318.5</td>
<td>567.4</td>
<td>871.7</td>
<td>761.3</td>
<td>1.195</td>
<td>0.131</td>
</tr>
</tbody>
</table>

* = $\pi + 2$ as limit when $\phi \rightarrow 0^\circ$. 

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Which Equations to Use

The bearing capacity can be obtained—often using empirical SPT or CPT data directly—to a sufficient precision for most projects.

The Terzaghi equations, being the first proposed, have been very widely used because of their greater ease of use. They are only suitable for a concentrically loaded footing on horizontal ground. Both the Meyerhof and Hansen methods are widely used. The Vesic method has not been much used.

<table>
<thead>
<tr>
<th>Use</th>
<th>Best for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terzaghi</td>
<td>Very cohesive soils where D/B &lt; 1 or for a quick estimate of quit to compare with other methods. Don’t use for footings with moments and/or horizontal forces or for tilted bases and/or sloping ground.</td>
</tr>
<tr>
<td>Hansen, Meyerhof, Vesic</td>
<td>Any situation that applies, depending on preference or familiarity with a particular method.</td>
</tr>
<tr>
<td>Hansen, Vesic</td>
<td>When base is tilted; when footing is on a slope or when D/B &gt; 1.</td>
</tr>
</tbody>
</table>
ADDITIONAL CONSIDERTIONS

1. A reduction factor is applicable for the third term of B effect as follows;

\[
r_r = 1 - 0.25 \log \left( \frac{B}{2} \right) \quad \text{for } B \geq 2 \text{ m}
\]

2. Which $\phi$

The soil element beneath the centerline of a strip footing is subjected to plane strain loading, and therefore, the plain strain friction angle must be used in calculating its bearing capacity, the plane strain friction angle can be obtained from a plane strain compression test. The loading condition of a soil element along the vertical centerline of a square or circular footing more closely resembles axisymmetric loading than plane strain loading, thus requiring a triaxial friction loading, which can be determined from a consolidated drained or undrained triaxial compression test.

Meyerhof proposed the corrected friction angle for use with rectangular footing as:

\[
\phi_{rectangular} = (1.1 - 0.1 \frac{B}{L}) \phi_{triaxial}
\]

Example: Compute the allowable bearing pressure using the Terzaghi equation for the footing and soil parameters shown.

Use a safety factor of 3 to obtain $q_a$?

Critical Thinking:

F=3 is recommended for cohesive soil

Solution:

Find the bearing capacity. ($B$ not known but $D$ is)

From equations and tables

$N_c=17.7$  $N_q=7.4$  $N_I=5.0$

$S_c=1.3$  $S_I=0.8$

\[
q_{ult} = c N_c S_c + \bar{q} N_q + 0.5 B \gamma N_I S_I
\]

$\bar{q}_{ult} = 20 \times 17.7 \times 1.3 + 17.30 \times 1.2 \times 7.4 + 0.5 \times B \times 17.3 \times 5.0 \times 0.8 = 613.8 + 34.6 B \text{ kPa}$

$q_a = q_{ult} / F$

$q_a = \frac{613.8 + 34.6B}{3} = 205 + 11.5B$

Likely $B$ ranges from 1.5 to 3m, with $B=3m$

\[
r_r = 1 - 0.25 \log \left( \frac{3}{2} \right) = 0.95
\]

$q_a = 205 + 11.5 \times 1.5 = 220 \text{ kPa}$

$q_a = 205 + 11.5 \times 3 \times 0.95 = 240 \text{ kPa} \quad \text{consider } q_a = 220 \text{ kPa}$
EFFECT OF WATER TABLE ON BEARING CAPACITY

The effective unit weight of the soil is used in the bearing-capacity equations for computing the ultimate capacity, in calculation for $\bar{q}$ in the $q_N$ term and in the term $0.5\gamma BN_y$.

1. When the water table is below the wedge zone [depth approximately $0.5B \tan (45 + \phi/2)$], the water table effects can be ignored for computing the bearing capacity.

2. When the water table lies within the wedge zone, if $B$ is known, computing the average effective unit weight to be used in the $0.5\gamma BN_y$ term will be as follows

$$\gamma_e = (2H - d_w) \frac{d_w}{H^2} \gamma_{wet} + \frac{\gamma'}{H^2} (H - d_w)^2$$

Where

- $H = 0.5 B \tan (45 + \phi/2)$
- $d_w = \text{depth of water table below base of footing}$
- $\gamma_{wet} = \text{wet unit weight of soil in depth } d_w$
- $\gamma' = \text{submerged unit weight of soil below water table } \gamma' = \gamma_{sat} - \gamma_w$

Example: for the square footing shown, what is the allowable bearing capacity?

Use $F=2$, Assume the soil above water table is wet with normal water content $w_N = 10\%$ and $G_s = 2.68$

Critical Thinking:

$F=2$ is recommended for cohesionless soil

Effective unit weight should be calculated

Solution:

$\phi = (1.1 - 0.1B/L) \phi_{tri} = 35°$

$H = 0.5 B \tan (45 + \phi/2) = 0.5 \times 2.5 \times \tan (45 + 35/2) = 2.4 \ m$

$d_w = 1.95 - 1.1 = 0.85$

$\therefore$ Water table lies within the wedge zone

$$\gamma_{dry} = \frac{\gamma_{wet}}{1 + w} = \frac{1}{1 + 0.1} \frac{18.10}{16.45} = 16.45 \ kN/m^3$$

$$V_s = \frac{G_s (9.81)}{2.68 (9.81)} = 0.626$$

$$V'_s = 1 - V_s = 1 - 0.626 = 0.374$$

The saturated unit weight is the dry weight + weight of water in voids

$$\gamma_{sat} = 16.45 + 0.374 (9.81) = 20.12 kN/m^3$$

$$\gamma_e = (2H - d_w) \frac{d_w}{H^2} \gamma_{wet} + \frac{\gamma'}{H^2} (H - d_w)^2$$

$$\gamma_e = (2 \times 2.4 - 0.85) \frac{0.85}{2.4^2} \frac{18.1}{18.1} + \frac{20.12 - 9.81}{2.4^2} (2.4 - 0.85)^2 = 14.85 \ kN/m^3$$
To calculate $\bar{q}$ in the $\bar{q}N_q$ term use $\gamma_{wet}$

$$\bar{q} = 18.1 \times 1.1 = 19.91 \text{ kN/m}^3$$

In the $0.5\gamma BN_b$ term use $\gamma_c = 14.85 \text{ kN/m}^3$

1. **Bearing Capacity by Terzaghi’s equations**  $\phi = 35$  $N_c = 57.8$  $N_q = 41.4$  $N_Y = 42.4$

$$q_{ult} = c N_c s_c + \bar{q} N_q + 0.5 B \gamma N_Y s_\gamma$$

$$r_\gamma = 1 - 0.25 \log (\frac{2.5}{2}) = 0.975$$

$$q_{ult} = 0 + 19.91 \times 41.4 + 0.5 \times 2.5 \times 14.85 \times 42.4 \times 0.8 \times 0.975 = 1438 \text{ kN/m}^2$$

$$q_o = q_{ult} / 2 = 1438 / 2 = 719 \text{ kN/m}^2$$

2. **Bearing Capacity by Meyerhof’s equations**  $\phi = 35$  $N_c = 46.1$  $N_q = 33.3$  $N_Y = 37.1$

$$q_{ult} = c N_c s_c d_c + \bar{q} N_q s_q d_q + 0.5 B \gamma N_Y s_\gamma d_\gamma$$

$$S_q = s_\gamma = 1 + 0.1 K_b B/L = 1 + 0.1 \times 3.7 \times 1 = 1.37$$

$$d_q = d_\gamma = 1 + 0.1 \sqrt{3.7} \times 1 = 1.08$$

$$q_{ult} = 19.91 \times 33.3 \times 1.37 \times 1.08 + 0.5 \times 2.5 \times 14.85 \times 37.1 \times 1.37 \times 1.08 \times 0.975 = 2000 \text{ kN/m}^2$$

$$q_o = q_{ult} / 2 = 2000 / 2 = 1000 \text{ kN/m}^2$$

3. **Bearing Capacity by Hansen’s equations**

$$q_o = q_{ult} / 2 = 1524 / 2 = 762 \text{ kN/m}^2$$

**BEARING CAPACITY FOR FOOTINGS ON LAYERED SOILS**

There are three general cases of the footing on a layered soil as follows:

**Case 1. Footing on layered clays (all $\phi = 0$).**  
- a. Top layer weaker than lower layer ($c_1 < c_2$)  
- b. Top layer stronger than lower layer ($c_1 > c_2$)  

$C_R$ = strength ratio

If $\frac{c_2}{c_1} = C_R \leq 0.7$

$$N_c = \frac{1.5d_1}{B} + 5.14C_R \leq 5.14 \text{ for strip footing}$$

$$N_c = \frac{3d_1}{B} + 6.05C_R \leq 6.05 \text{ for circular & square footing}$$

When $0.7 \leq C_R \leq 1$  Reduce the aforesaid $N_c$ by 10%

If $\frac{c_2}{c_1} = C_R \geq 1$

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\[ N_{c1} = 4.14 + \frac{0.5B}{d_1} \quad \quad N_{c2} = 4.14 + \frac{1.1B}{d_1} \quad \text{for strip footing} \]

\[ N_{c1} = 5.05 + \frac{0.33B}{d_1} \quad \quad N_{c2} = 5.05 + \frac{0.66B}{d_1} \quad \text{for round & square footing} \]

Notice: When the top layer is very soft with a small \( d_1/B \) ratio, one should give consideration either to placing the footing deeper onto the stiff clay or to using some kind of soil improvement method

Case 2. Footing on layered \( \phi-c \) soils with \( a, b \) same as case 1.
A modified angle of friction and soil cohesion may be used to determine the bearing capacity when the depth of the top layer \( (d_1) \) under the base of foundation is less than \( H \).
Calculate \( H \):
\[ H = 0.5B \tan(45 + \frac{\phi}{2}) \]
IF \( H > d_1 \) then compute
\[ \phi' = \frac{d_1 \phi_1 + (H - d_1)\phi_2}{H} \]
\[ c' = \frac{d_1 c_1 + (H - d_1)c_2}{H} \]

For case 1 and 2, if the top layer is soft the calculated bearing capacity should not be more than \( q_{ult} = 4c_1 + q \)

Case 3. Footing on layered sand and clay soils.
a. Sand overlying clay
b. Clay overlying sand

IF \( H > d_1 \) then
1. Find \( q_{ult} \) based on top-stratum soil parameters using one of bearing capacity equations.
2. Assume a punching failure bounded by the base perimeter of dimensions \( B \times L \). Here include the \( q \) contribution from \( d_1 \), and compute \( q'_{ult} \) of the lower stratum using the base dimension \( B \).
3. Compare \( q_{ult} \) to \( q'_{ult} \) and use the smaller value.

Example: A footing of \( B = 3 \), \( L = 6 \) m is to be placed on a two-layer clay deposit a:
Estimate the ultimate bearing capacity using Hansen’s equation?

Critical Thinking:
Footing on layered caly soil \( \phi=0 \)
Solution:
Check H ; H = 0.5 B tan (45 + \phi/2)

H=0.5 × 3 tan 45° = 1.5 m > (d_1=1.22 m)

\[ C_R = \frac{C_2}{C_1} = \frac{115}{77} = 1.5 > 1 \]

N_{c1}= 4.14 +0.5x3/1.22= 5.37
N_{c2}= 4.14 + 1.1x3/1.22= 6.84

\[ N_c = \frac{N_1 \times N_2}{N_1 + N_2} \times 2 = \frac{5.37 \times 6.84}{5.37 + 6.84} \times 2 = 6 \]

\[ q_{ult} = 5.14 s_u (1 + s'_c + d'_c - i'_c - b_c - g_c) + \bar{q} \]

q_{ult} = 6 × 77 (1 + (0.2x3/6) + (0.4x1.83/3) ) + 17.26 x1.83 x1x1 = 652 kPa

BEARING CAPACITY FOR FOOTINGS WITH ECCENTRIC LOADING

Two methods can be followed to solve this case
Method 1. Use either the Hansen or Vesic bearing-capacity equation with the following adjustments:
   a. Use B’ in the yBNy term.
   b. Use B’ and L’ in computing the shape factors.
   c. Use actual B and L for all depth factors.

\[ q_a = \frac{q_{ult}}{F} \quad \text{and} \quad P_a = q_a B' L' \]

Method 2. Use the Meyerhof general bearing-capacity equation and a reduction factor Re used as

\[ q_{ult, \text{des}} = q_{ult, \text{comp}} \times Re \]

Re = 1 - 2e/B \quad (cohesive soil)

Re = 1 - \sqrt{e/B} \quad (cohesionless soil and for 0 < e/B < 0.3)

Alternatively, one may directly use the Meyerhof equation with B’ and L’ used to compute the shape and depth factors and B’ used in the 0.5γB’N_y term.
**Example:** A square footing is 1.8 x 1.8 m with a 0.4 x 0.4 m square column. It is loaded with an axial load of 1800 kN and $M_x = 450$ kN m; $M_y = 360$ kN m. Undrained triaxial tests (soil not saturated) give $\phi = 36^\circ$ and $c = 20$ kPa. The footing depth $D = 1.8$ m; the soil unit weight $\gamma = 18.00$ kN/m$^3$; the water table is at a depth of 6.1 m from the ground surface. What is the allowable soil pressure, if $F = 3.0$, using the Hansen bearing-capacity equation with $B'$, $L'$, Meyerhof's equation; and the reduction factor $R_e$?

**Solution:**

- $e_x = \frac{M_y}{V} = \frac{360}{1800} = 0.2 < \frac{B}{6}$
- $e_y = \frac{M_x}{V} = \frac{450}{1800} = 0.25 < \frac{L}{6}$
- $B_{\min} = 4e_x + w_x = 4 \times 0.2 + 0.4 = 1.2 < 1.8$
- $L_{\min} = 4e_y + w_y = 4 \times 0.25 + 0.4 = 1.4 < 1.8$
- $B' = B - 2e_x = 1.8 - 2 \times 0.2 = 1.4$ m
- $L' = L - 2e_y = 1.8 - 2 \times 0.25 = 1.3$ m

Hansen's bearing-capacity equation

$N_c = 51$  $N_q = 38$  $N_y = 40$

Compute shape factors using $B', L'$

$s_c = 1 + \left( \frac{N_q}{N_c} \right) \left( \frac{B'}{L'} \right) = 1.69$

$s_q = 1 + \left( \frac{B'}{L'} \right) \sin \phi = 1.55$

$s_y = 1 - 0.4B'/L' = 0.6 \phi$

Compute depth factors using $B, L$

$d_c = 1 + 0.4D/B = 1.4$

$d_q = 1 + 2 \tan \phi (1 - \sin \phi) \frac{D}{B} = 1.25$

$d_y = 1$

$q_{ult} = c N_c s_c d_c + qN_q s_q d_q + 0.5 B' \gamma N_y s_y d_y$

$= 20 \times 51 \times 1.69 \times 1.4 + 18 \times 1.8 \times 38 \times 1.55 \times 1.25 + 0.5 \times 1.4 \times 18 \times 40 \times 0.6 \times 1 = 5100$ kPa

$q_a = 5100/3 = 1700$ kPa

**The actual soil pressure is 1800/(1.4\times1.3) = 989$ kPa**
Meyerhof’s equation

\[ K_p = \tan^2 (45 + \frac{\phi}{2}) = 3.85 \]
\[ N_c = 51 \quad N_q = 38 \quad N_r = 44 \]
\[ s_c = 1 + (0.2K_p B/L) = 1.77 \]
\[ s_q = 1 + (0.1K_p B/L) = 1.39 \]
\[ d_c = 1 + (0.2\sqrt{K_p D/B}) = 1.39 \]
\[ d_q = 1 + (0.1\sqrt{K_p D/B}) = 1.20 \]

\[ q_{ult} = 20 \times 51 \times 1.77 \times 1.39 + 18 \times 1.8 \times 38 \times 1.39 \times 1.2 + 0.5 \times 18 \times 1.8 \times 44 \times 1.39 \times 1.2 = 5752 \text{ kPa} \]

\[ R_e = 1 - \frac{2e}{B} \quad (\text{cohesive soil}) \]
\[ R_{ex} = 1 - \frac{2 \times 0.2}{1.8} = 0.78 \quad R_{ey} = 1 - \frac{2 \times 0.25}{1.8} = 0.72 \]
\[ q_{ult} = 5752 \times 0.78 \times 0.72 = 3230 \text{ kPa} \]
\[ q_a = \frac{3230}{3} = 1073 \text{ kPa} \]

**The actual soil pressure is 1800/ (1.8x1.8) = 555 kPa**

\[ R_e = 1 - \sqrt{\frac{e}{B}} \quad (\text{cohesionless soil}) \quad (\text{This can be used since the cohesion of the soil is low (20)} \]
\[ R_{ex} = 1 - \sqrt{\frac{0.2}{1.8}} = 0.67 \quad R_{ey} = 1 - \sqrt{0.25/1.8} = 0.63 \]
\[ q_{ult} = 5752 \times 0.67 \times 0.63 = 2428 \text{ kPa} \]
\[ q_a = \frac{2428}{3} = 809 \text{ kPa} \]

**BEARING CAPACITY FOR FOOTINGS WITH INCLINED LOAD**

Inclined loads are produced when the footing is loaded with both a vertical \( V \) and a horizontal component(s) \( H_i \) of loading (refer to Table 4-5 and its figure). This loading is common for many industrial process footings where horizontal wind loads are in combination with the gravity loads. In any case the load inclination results in a bearing capacity reduction over that of a foundation subjected to a vertical load only.

**Safety Factor Against Sliding**

The footing must be stable against both sliding and bearing in case it subjected to a horizontal load component, for sliding the general equation for the factor of safety is;

\[ F_S = \frac{V \tan \delta + C_s B' L' + P_p}{H} \]

**Example:** The data obtained by the load test is \( V_{ult} = 1060 \text{ kN} \)
\( H_{ult} = 382 \text{ kN} \)
Find the ultimate bearing capacity by Hansen and Vesic’s equations?

**Critical Thinking:**

Change \( \phi_{\text{triaxial}} \) to \( \phi_{\text{plane strain}} \)

---

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Solution:

\( \phi_{\text{plane strain}} = (1.1 - 0.1B/L) \phi_{\text{triaxial}} = (1.1 - 0.1 \times 0.5/2)43^\circ = 47^\circ \)

Hansen’s Bearing Capacity Equation

\[
q_{\text{ult}} = cNcsydyc \gamma y + \bar{q}Nqsdq \gamma q + qq + 0.5 \gamma N\gamma d\gamma i\gamma \gamma b\gamma
\]

From Equations;

\( N_q = 187 \quad N_r = 300 \)

\[
d_{QB} = 1 + 2 \tan(1 - \sin \phi)^2 \frac{D}{B} = 1.155
\]

\[
d_{QL} = 1 + 2 \tan(1 - \sin \phi)^2 \frac{D}{L} = 1.04
\]

\[
dy_B = dy_L = 1
\]

\[
i_B = (1 - (0.5H_B / (V + A_t C_a Cot \phi)))^{2.5} = 1
\]

\[
i_{yB} = (1 - (0.7H_B / (V + A_t C_a Cot \phi)))^{3.5} = 1
\]

\[
i_{QL} = (1 - (0.5H_L / (V + A_t C_a Cot \phi)))^{2.5} = 1 - 0.5 \times 382/1060
\]

\[
i_{yL} = (1 - (0.7H_L / (V + A_t C_a Cot \phi)))^{3.5} = 0.608
\]

\[
S_{QB} = 1 + \sin \phi B' i_{QB} / L' = 1.18
\]

\[
S_{QL} = 1 + \sin \phi L' i_{QL} / B' = 2.78
\]

\[
S_{yB} = 1 - 0.4 B' i_{yB} / (L' i_{yL}) = 0.723
\]

\[
S_{yL} = 1 - 0.4 L' i_{yL} / (B' i_{yB}) = 0.422 \quad \text{Use 0.6 both should be equal or more than 0.6}
\]

Substitute the factors obtained for each direction in Hansen’s equation and take the smaller ultimate bearing capacity

\[
q_{\text{ult}} = 9.43 \times 0.5 \times 187 \times 1.18 \times 1.16 \times 1 + 0.5 \times 0.5 \times 9.43 \times 300 \times 0.732 \times 1 \times 1 = 1700 \text{ kPa}
\]

\[
q_{\text{ult}} = 9.43 \times 0.5 \times 187 \times 2.78 \times 1.04 \times 0.608 + 0.5 \times 2.0 \times 9.43 \times 300 \times 0.6 \times 1 \times 0.361 = 2150 \text{ kPa}
\]

Vesic’s Bearing Capacity Equation

\[
q_{\text{ult}} = cNcsydyc \gamma y + \bar{q}Nqsdq \gamma q + qq + 0.5 \gamma N\gamma d\gamma i\gamma \gamma b\gamma
\]

From Equations

\( N_q = 187 \quad N_r = 404 \)

\[
S_y = 1 + B/L \tan \phi = 1 + (0.5/2) \tan 47 = 1.27
\]

\[
S_y = 1 - 0.4 B/L = 1 - 0.4 (0.5/2) = 0.9 \geq 0.6 \quad \text{OK}
\]

\[
d_q = 1 + 2 \tan \phi (1 - \sin \phi)^2 k = 1.16
\]

\[
d_y = 1
\]

\[
m = (2 + (L/B)) / (1 + (L/B))
\]

\[
i_{QL} = (1 - (H_L / (V + A_t C_a Cot \phi)))^m = (1 - 382/1060)^{1.2} = 0.585
\]

\[
i_{yL} = (1 - (H_L / (V + A_t C_a Cot \phi)))^{m+1} = 0.374
\]

\[
q_{\text{ult}} = 9.43 \times 0.5 \times 187 \times 1.27 \times 1.16 \times 0.585 + 0.5 \times 0.5 \times 9.43 \times 404 \times 0.9 \times 1 \times 0.374 = 1080 \text{ kPa}
\]
BEARING CAPACITY OF FOOTINGS ON SLOPES

Example: A 2 x 2 m square footing has a ground slope of $\beta = 0^\circ$ and base slope of $\eta = 10^\circ$. Are the footing dimensions adequate for the given load with a factor of safety 3?
Use Hansen’s and Vesić’s equations

Critical Thinking:
Assume angle of friction between concrete and soil ($\delta = \phi = 25^\circ$

Base adhesion $C_a = 0.6$ to $1 \times C = 25$ kPa

Neglect passive earth pressure for $D = 0.3$ m

Solution:
Factor of safety against sliding must be checked

$$F_s = \frac{V \tan \delta + C_a B'L' + P_r}{H} = \frac{600 \tan 25 + 25 \times 2 \times 2}{200} = 1.9$$

Hansen’s Bearing Capacity Equation

$$q_{ult} = c \left[ N_c s_d c_i g g_b c + \bar{q} N_q s_d q_i q g b_q + +0.5 B \gamma N_y s_r d_r i_r g_y b_y \right]$$

From table $N_c = 20.7$    $N_q = 10.7$    $N_y = 6.8$

$$d_c = 1 + 0.4 D/B = 1.06$$
$$d_q = 1 + 2 \tan \phi (1 - \sin \phi)^2 D/B = 1.05$$
$$d = 1$$

$$i_{qB} = (1 - (0.5H_b / (V + A_f \ C_a \ Cot \phi)))^3 = 0.675$$
$$i_{kB} = (1 - ((0.7 - \eta/450)H_b / (V + A_f \ C_a \ Cot \phi)))^4 = 0.481$$
$$i_{LY} = 1$$
$$i_c = i_q - (1-i_q)/(N_q-1) = 0.641$$

$$S_{CB} = 1 + (N_q/N_c)(B' i_{qB} / L) = 1.329$$
$$S_{QB} = 1 + \sin \phi B' i_{qB} / L = 1.285$$
$$S_{yB} = 1 - 0.4 B' i_{yB} / L i_{yL} = 0.808$$

$$\eta = 10^\circ = 0.175 \text{ rad}$$

$$b_{cB} = 1 - \eta/147 = 0.93$$
$$b_{qB} = \exp(-2\eta \tan \phi) = 0.849$$
$$b_{yB} = \exp(-2.7\eta \tan \phi) = 0.802$$

Ground factors $g_i = 1$ for $\beta = 0$

$$q_{ult} = 25 \times (20.7 \times 1.329 \times 1.06 \times 0.641 \times 0.93 + 17.5 \times 0.3 \times 1.285 \times 1.05 \times 0.675 \times 0.848 \times 0.5 \times 17.5 \times 2 \times 0.808 \times 1 \times 0.481 \times 0.802) = 515 \text{ kPa}$$

$$P_a = A \times q_a = 2 \times 2 \times 515/3 = 686 \text{ kN} > 600 \quad \text{OK}$$

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BEARING CAPACITY FROM IN SITUE SPT

\[ q_a = \frac{N}{F_1} K_d \quad B \leq 1.2 \text{ m} \]

\[ q_a = \frac{N}{F_2} \left( \frac{B + 0.3}{B} \right)^2 K_d \quad B > 1.2 \text{ m} \]

\[ q_a = \frac{N}{F_2} K_d \quad \text{For mat foundation} \]

- \( q_a \): allowable bearing pressure for \( \Delta H_o = 25\)-mm or 1-in. settlement, kPa, for any other value of settlement \( \Delta H_i \), \( q_a = \Delta H_i / \Delta H_o \times q_a \)
- \( K_d = 1 + 0.33 D/B \leq 1.33 \)
- \( N = \) the statistical average value of blows for the footing influence zone of about 0.5B above footing base to at least 2B below. To convert \( N'70 \times 70 = N_{55} \times 55 \)

**Example:** Find the allowable bearing pressure for a soil of \( N_{70} = 24 \), if the foundation depth = 1m and width = 3m for a tolerable settlement of 15 mm?

**Solution:**

\[ q_a = \frac{24}{0.06} \left( \frac{3 + 0.3}{3} \right)^2 \left( 1 + 0.33 \frac{1}{3} \right) = 537 \text{ kPa} \]

\( q_a \) for 15mm settlement= 537 x 15/25 = 322 kPa

BEARING CAPACITY FROM IN SITUE CPT

An approximation for the bearing capacity should be applicable for \( D/B < 1.5 \) from an averaged value of \( q_c \) (cone point resistance in kg/cm\(^2\)) over the depth interval from about \( B/2 \) above to 1.1B below the footing base. For cohesionless soils one may use

\[ q_{ult} = 28 - 0.0052(300 - q_c)^{1.5} \text{ kg/cm}^2 \quad \text{strip} \]

\[ q_{ult} = 48 - 0.009(300 - q_c)^{1.5} \text{ kg/cm}^2 \quad \text{square} \]

For cohesive soils one may use

\[ q_{ult} = 2 + 0.28q_c \text{ kg/cm}^2 \quad \text{strip} \]

\[ q_{ult} = 5 + 0.34q_c \text{ kg/cm}^2 \quad \text{square} \]
5. FOUNDATION SETTLEMENTS

Settlement of Shallow Foundation in Cohesive Soil

The stress change (\(\Delta q\)), from an added load, produces a time-dependent accumulation of particle rolling, sliding, crushing, and elastic distortions in a limited influence zone beneath the loaded area. The statistical accumulation of movements in the direction of interest is the settlement. In the vertical direction the settlement will be defined as \(\Delta H\).

The principal components of \(\Delta H\) are particle rolling and sliding, which produce a change in the void ratio, and grain crushing, which alters the material slightly. Only a very small fraction of \(\Delta H\) is due to elastic deformation of the soil grains. As a consequence, if the applied stress is removed, very little of the settlement \(\Delta H\) is recovered. Even though \(\Delta H\) has only a very small elastic component, it is convenient to treat the soil as a pseudo-elastic material with "elastic" parameters \(E_s\), \(G'\), \(\mu\) and \(k_s\) to estimate settlements.

There are two major problems with soil settlement analyses:

1. Obtaining reliable values of the "elastic" parameters. Problems of recovering "undisturbed" soil samples mean that laboratory values are often in error by 50 percent or more.
2. Obtaining a reliable stress profile from the applied load. We have the problem of computing both the correct numerical values and the effective depth \(H\) of the influence zone. Theory of Elasticity Equations are usually used for the stress computations, with the influence depth \(H\) below the loaded area taken from \(H = 0\) to \(H \to \infty\) (but more correctly from \(0\) to about \(4B\) or \(5B\)).

The values from these two problem areas are then used in an equation of the general form

\[
\Delta H = \int_0^H \varepsilon dH
\]

Settlements are usually classified as follows:

1. **Immediate Settlement:** Immediate settlement, also known as distortion settlement, initial settlement, or elastic settlement, occurs immediately upon the application of the load, due to lateral distortion of the soil beneath the footing. In clays, where drainage is poor, it is reasonable to assume that immediate settlement takes place under undrained conditions where there is no volume change. The average immediate settlement under a flexible footing generally is estimated using the theory of elasticity. Immediate settlement analyses are used for all fine-grained soils including silts and clays with a degree of saturation \(S \leq 90\) percent and for all coarse-grained soils with a large coefficient of permeability, say, above \(10^{-3}\) m/s.

2. **Primary Consolidation Settlement:** is a time-dependent process in saturated clays, where the foundation load is gradually transferred from the pore water to the soil skeleton. Immediately after loading, the entire applied normal stress is carried by the water in the voids, in the form of excess pore water pressure. With time, the pore water drains out into the more porous granular soils at the boundaries, thus dissipating the excess pore water pressure and increasing the effective stresses. Depending on the thickness of the clay layer, and its consolidation characteristics, this process can take from a few days to several years. Consolidation settlement analyses are used for all saturated, or nearly saturated, fine-grained soil. For these soils we want estimates of both settlement \(\Delta H\) and how long a time it will take for most of the settlement to occur.
3. **Secondary compression settlement**: takes place at constant effective stress, when there is no more dissipation of excess pore water pressure. For simplicity, it is assumed to start occurring when the primary consolidation is completed at time $t_p$, and the settlement increases linearly with the logarithm of time.

**Immediate Settlement $S_i$**

Theory of elasticity is used to calculate the settlement at the corner of a rectangular base of $B' \times L'$ by the equation;

$$S_i = q_o B \left( \frac{1 - \mu^2}{E_s} \right) m \left( \frac{1 - 2\mu}{1 - \mu} \right) I_F$$

Where

- $S_i$ = the immediate elastic settlement
- $q_o$ = intensity of contact pressure in same unit of $E_s$
- $B'$ = least lateral dimension of contributing base area in unit of $S_i$
- $E_s, \mu$ = elastic soil properties
- $I_i$ = influence factors, depends on size of footing, depth, thickness of layer $H$, and $\mu$.
- $m$ = no. of corners contributing to settlement $S_i$

**Notice:**

1. The influence depth $H$: the smaller of $5B$ or the depth to the hard layer, where the "hard" layer was defined as that where $E_h > 10 \times E_s$ of the next adjacent layer.
2. $S_i$ $_r_i_g_i_d \approx 0.93$ $S_i$ $f_l_e_x_i_b_l_e$ $c_e_n_t_e_r$
3. Obtain the weighted average soil properties if the foundation influence depth ($H$) is of $n$ layers

$$E_{av} = \frac{E_1 H_1 + E_2 H_2 + E_3 H_3}{H}$$

4. To find $I_F$ from the graph use actual dimensions of the footing $L, B, D$.

$$I_1 = \frac{1}{\pi} \left[ M \ln \left( \frac{1 + \sqrt{M^2 + 1}}{\sqrt{M^2 + N^2 + 1}} \right) + \ln \left( \frac{M + \sqrt{M^2 + 1}}{M + \sqrt{M^2 + N^2 + 1}} \right) \right]$$

$$I_2 = \frac{N}{2\pi} \tan^{-1} \left( \frac{M}{N\sqrt{M^2 + N^2 + 1}} \right) \quad \text{tan}^{-1} \text{ in radians} \quad \text{where } M = \frac{L'}{B'} \quad N = \frac{H}{B'}$$

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Values of $I_1$ and $I_2$ to compute the Steinbrenner influence factor $I_r$ for use in Eq. (5-16a) for several $N = H/B'$ and $M = L/B$ ratios

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<td>0.112</td>
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TABLE 5-6
Equations for stress-strain modulus $E_s$ by several test methods
$E_s$ in kPa for SPT and units of $q_c$ for CPT; divide kPa by 50 to obtain ksf. The $N$ values should be estimated as $N_{50}$ and not $N_{60}$. Refer also to Tables 2-7 and 2-8.

<table>
<thead>
<tr>
<th>Soil</th>
<th>SPT</th>
<th>CPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand (normally consolidated)</td>
<td>$E_s = 500(N + 15)$</td>
<td>$E_s = (2 \text{ to } 4)q_c$.</td>
</tr>
<tr>
<td></td>
<td>$= 7000 \sqrt{N}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 6000N$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{2E_s}{(5000 \text{ to } 22000)} \cdot \ln N$</td>
<td></td>
</tr>
<tr>
<td>Sand (saturated)</td>
<td>$E_s = 250(N + 15)$</td>
<td>$E_s = F_q e$</td>
</tr>
<tr>
<td>Sands, all (norm. consel.)</td>
<td>$\frac{1}{E_s} = (2600 \text{ to } 2900)N$</td>
<td>$e = 1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = 3.5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$e = 0.6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F = 7.0$</td>
</tr>
<tr>
<td>Sand (overconsolidated)</td>
<td>$\frac{1}{E_s} = 40000 + 1050N$</td>
<td>$E_s = (6 \text{ to } 30)q_c$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravelly sand</td>
<td>$E_s = 1200(N + 6)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 600(N + 6)$</td>
<td>$N \leq 15$</td>
</tr>
<tr>
<td></td>
<td>$= 600(N + 6) + 2000$</td>
<td>$N &gt; 15$</td>
</tr>
<tr>
<td>Clayey sand</td>
<td>$E_s = 320(N + 15)$</td>
<td>$E_s = (3 \text{ to } 6)q_c$.</td>
</tr>
<tr>
<td>Silts, sandy silt, or clayey silt</td>
<td>$E_s = 300(N + 6)$</td>
<td>$E_s = (1 \text{ to } 2)q_c$.</td>
</tr>
</tbody>
</table>

If $q_c < 2500$ kPa use $E_s = 2.5q_c$.
If $2500 < q_c < 5000$ use $E_s = 4q_c + 5000$.

$E'_s =$ constrained modulus $= \frac{E_s(1 - \mu)}{(1 + \mu)(1 - 2\mu)} = \frac{1}{n_s}$

Soft: clay or clayey silt

For point $O$: use $4 \times Oabc$. For point $O'$: use $O'e'bd$.

(a) Square loaded area $= O'e'bd$. (b) Rectangle with loaded area $= O'gbd$.

For point $O$: use $Oabc + Ode + Oe'bf + O'ga$.
For point $O'$: use $O'gbd$.

(c) Point outside loaded area $= dfgy$.
For point $O$: use $Oghb = Oghf - Oghc - Oabc + Obe$. For point $O'$: use $skidgm$.

(d) For loaded area $= kildgm$.
For point $O$: $Oibe + Oagf + Ofml + O'bf + O'fj + O'e'bd$.
Example: Soil data and profile are shown. A best average of N values (SPT) gave \( N'_{70} = 20 \). Column loads including dead and live loads are estimated in the range of 450 to 900 kN. Recommend \( q_a \) for this project so that \( \Delta H \) is limited to not over 25 mm?

Solution:

From relation with standard penetration test
\[
q_a = \frac{N_{70}}{0.06} \left( \frac{B+0.3}{B} \right)^2 (1 + 0.33 \frac{D}{B}) \quad B > 1.2 \ m
\]

For \( B=1.5 \text{ m} \), \( q_a = 640 \text{ kPa} \) for \( B=2 \text{ m} \), \( q_a = 550 \text{ kPa} \)

Actual \( q \) if \( B=1.5 \text{ m} \) \( q = 900/(1.5 \times 1.5) = 400 \text{ kPa} \) \( q = 450/2.25 = 200 \text{ kPa} \)

The maximum actual pressure is less than the allowable bearing pressure recommended for 25 mm settlement.

Now, assume the allowable bearing capacity = 300 kPa \( A=900/300=3 \text{ m}^2 \) \( B=1.7 \text{ m} \)

Calculation of settlement
\[
q_o = 900/ (1.7 \times 1.7) = 310 \text{ kPa}
\]

Find soil parameters (Table 5-6), \( N_{55} = N_{70} \times 70/55 = 20 \times 70/55 = 25 \)
\( \mu = 0.3 \)
\( E_s = 500(N_{55}+15) = 500 (25+15) = 20000 \text{ kPa} \)
\( E_s = 7000\sqrt{N_{55}} = 7000\sqrt{25} = 35000 \text{ kPa} \)
\( E_s = 2600 \text{ N}_{55} = 2600(25) = 65000 \text{ kPa} \)

Use \( E_s = 20000 \text{ kPa} \) as a conservative value

\[
S_i = q_o B \left( \frac{1 - \mu^2}{E_s} \right) m \left( I_1 + \frac{1 - 2\mu}{1 - \mu} I_2 \right) I_F
\]

Settlement at the center of the footing
\( m=4 \)
\( B' = 1.7/2 = 0.85 \text{ m} \)

To find \( I_1, I_2 \)
\( N=H/B' = 8/0.85 = 9.4 \quad M=L'/B' = 1 \quad I_1 = 0.491 + (0.4) \times (0.007) = 0.494, \quad I_2 = 0.0164 \)

To find \( I_F \)
\( L/B=1, \quad \mu = 0.3, \quad D/B= 1.5/1.7= 0.9 \quad \text{from } I_F \text{ curves } \quad I_F = 0.67 \)

\[
S_i = 310 \times 0.85 ((1-0.3^2)/20000) \times 4 \times (0.494+(1-2x0.3)/(1-0.3)) \times 0.0164 \times 0.67 = 0.01617 \text{ m} = 16.17 \text{ mm}
\]
Settlement at the corner of the footing

M=1
B'= 1.7m
To find \( I_1, I_2 \)
N=H/B' = 8/1.7 = 4.7 \( I_1 = 0.408+(0.7)\times(0.029)=0.428, \) \( I_2 = 0.033 \)

To find \( I_F \)
L/B=1, \( \mu = 0.3, \) D/B= 1.5/1.7= 0.9 \(__\) from \( I_F \) curves ---- \( I_F = 0.67 \)
\( S_i = 310 \times 1.7 \frac{(1-0.3^2)}{20000} \times 1 \times (0.428+(1-2\times0.3)/(1-0.3) \times 0.033) \times 0.67 = 0.0069 \text{ m} \)
\( =6.9 \text{ mm} \)

**Example:** Estimate the settlement of the center of raft (mat) foundation for a building, with given Data are as follows:
\( q_0 = 134 \text{ kPa} \) \( \text{ B X L} = 33.5 \times 39.5 \text{ m} \) measured \( \Delta H = \text{ about 18 mm} \)
Soil is layered clays with one sand seam from ground surface to sandstone bedrock at - 14 m, mat at -3 m.
\( E_s \) from 3 to 6 m = 42.5 MPa,
\( E_s \) from 6 to 14 m = 60 MPa, \( \mu=0.35 \)
\( E_s \) for sandstone > 500 MPa

**Critical Thinking:** find the average value of soil properties
\[ E_{av} = \frac{E_1H_1+E_2H_2+E_3H_3}{H} \]

**Solution:**
\( E_s \) average= \( (42.5 \times 3 + 60\times8)/11 = 55 \text{ MPa} \)

H=11m, \( m =4, \) \( B' =33.5/2 = 16.75\text{m}, \) \( N=H/B'=11/16.75= 0.66 \)

M= \( L/B = (39.5/2) / (33.5/2)= 1.18 \) \( D/B=3/33.5=0.09 \)
\( I_1 = 0.075 \) \( I_2 = 0.085 \) \( I_F = 0.85 \)

\[ S_i = q_o B \left( \frac{1-\mu^2}{E_s} \right) m \left( I_1 + \frac{1-2\mu}{1-\mu} I_2 \right) I_F \]
\( S_i = 134 \times 16.75 \frac{(1-0.35^2)}{55000} \times 4 \times (0.075+(1-2\times0.35)/(1-0.35) \times 0.085) \times 0.85 = 0.0140 \text{ m} \)
Consolidation Settlement $S_c$

Consolidation of a soil will be defined as a void -ratio reduction which takes place as a function of time. It is considered to be taking place during the time excess pore pressure exits in the consolidating stratum owing to an increase in pressure within the stratum from exterior loading. This consolidation is also termed primary consolidation. Secondary consolidation is that additional void- ratio change (settlement) taking place after the excess pore pressure has essentially dissipated. This consolidation is sometimes termed creep and may continue for some considerable time after primary consolidation is complete. This is especially true in organic or peat soils for most soils secondary consideration is much less than “consolidation” settlement except for the organic or peat soils mentioned. Settlement due to consolidation may be predicted by the one dimensional consolidation theory.

In a clay layer with an initial thickness of $H$ and a void ratio of $e_0$, the final consolidation settlement $S_c$ due to the applied pressure $q$ can be estimated from

$$S_c = H \frac{\Delta e}{1 + e_0}$$

Where $\Delta e$ is the change in the void ratio due to the applied pressure $q$. $H_0$ and $e_0$ can be obtained from the soil data, and $\Delta e$ has to be computed as follows.

Three different cases, as shown in figure below, are discussed here. Point I corresponds to the initial state of the clay, where the void ratio and the vertical stress are $e_0$ and $\sigma_{vo}$, respectively. With the vertical stress increase of $\Delta \sigma_v$, consolidation takes place, and the void ratio decreases by $\Delta e$. Point F corresponds to the final state, at the end of consolidation. Point P corresponds to the preconsolidation pressure $\sigma_p$ on the virgin consolidation line.
Case I. If the clay is normally consolidated, $S_c$ can be computed from:

$$S_c = \frac{H}{1 + e_o} C_c \log\left(\frac{\sigma_{vo} + \Delta\sigma_v}{\sigma'_vo}\right)$$

Case II. If the clay is overconsolidated and $\sigma_{vo} + \Delta\sigma_v \leq \sigma_p$ (i.e., the clay remains overconsolidated at the end of consolidation), $S_c$ can be computed from:

$$S_c = \frac{H}{1 + e_o} C_r \log\left(\frac{\sigma_{vo} + \Delta\sigma_v}{\sigma'_vo}\right)$$

Case III. If the clay is overconsolidated and $\sigma_{vo} + \Delta\sigma_v \geq \sigma_p$ (i.e., the clay becomes normally consolidated at the end of consolidation), $S_c$ can be computed from:

$$S_c = \frac{H}{1 + e_o} \left\{ C_r \log\left(\frac{\sigma_p}{\sigma_{vo}}\right) + C_c \log\left(\frac{\sigma_{vo} + \Delta\sigma_v}{\sigma'_vo}\right) \right\}$$

$C_c$: Compression Index; is the slope of the normal consolidation line in a plot of the logarithm of vertical effective stress verses void ratio.

$C_r$: Recompression Index; is the average slope of the unloading/reloading curves in a plot of the logarithm of vertical effective stress verses void ratio.

Example: The soil profile at a site for a proposed office building consists of a layer of fine Sand 10.4 m thick above a layer of soft normally consolidated clay 2 m thick. Below the soft clay is a deposit of coarse sand. The groundwater table was observed at 3 m below ground level. The void ratio of the sand is 0.76 and the water content of the clay is 43%. The building will impose a vertical stress increase of 140 kPa at the middle of the clay layer. Estimate the primary consolidation settlement of the clay. Assume the soil above the water table to be saturated, $C_c 0.3$ and $G_s = 2.7$.

Critical Thinking: For normally consolidated clay, the appropriate equation is that for case I

Solution:

$$S_c = H \frac{\Delta e}{1 + e_o} = H \frac{1}{1 + e_o} C_c \log\left(\frac{\sigma_{vo} + \Delta\sigma_v}{\sigma'_vo}\right)$$

Calculate the current effective stress and void ratio at the middle of the clay layer

Sand layer

$$\gamma_{sat} = \frac{W}{V} = \left(\frac{G_s + S e}{1 + e}\right) \gamma_w = \frac{2.7 + 0.76}{1 + 0.76} \times 9.81 = 19.3 \text{ kN/m}^3$$

$$\gamma' = \gamma_{sat} - \gamma_w = 19.3 - 9.8 = 9.5 \text{ kN/m}^3$$

2015 - 2016
Clay layer
$S e_o = w G_s = 0.43 \times 2.7 = 1.16$
$\gamma' = 7.7 \text{ kN/m}^3$

Effective stresses
$\sigma'_{vo} = 19.3 \times 3 + 9.5 \times 7.4 + 7.7 \times 1 = 135.9 \text{ kPa}$
$\sigma'_{fin} = \sigma'_{vo} + \Delta \sigma = 135.9 + 140 = 275.9 \text{ kPa}$

$S_c = 2/ (1 + 1.16) \times 0.3 \log (275.9/ 135.9) = 0.0854 \text{ m} \quad 85.4 \text{ mm}$

**Example:** Assume the same soil stratigraphy and soil parameters as in previous example except that the clay has an overconsolidation ratio of 1.5, $w=38\%$, $C_v=0.05$. Determine the primary consolidation settlement of the clay?

**Critical Thinking:** Since the soil is overconsolidated, you will have to check whether the preconsolidation stress is less than or greater than the sum of the current vertical effective stress and the applied vertical stress at the center of the clay. This check will determine the appropriate equation to use.

Solution:
Clay layer
$S e_o = w G_s = 0.38 \times 2.7 = 1.03$
$\gamma' = 8.2 \text{ kN/m}^3$

Effective stresses
$\sigma'_{vo} = 19.3 \times 3 + 9.5 \times 7.4 + 8.2 \times 1 = 136.4 \text{ kPa}$
$\sigma'_{fin} = \sigma'_{vo} + \Delta \sigma = 136.4 + 140 = 276.4 \text{ kPa}$

Preconsolidation stress $\sigma_p = 1.5 \times 136.4 = 204.6 \text{ kPa} < \sigma'_{vo} + \Delta \sigma$

$S_c = \frac{H}{1 + e_o} \{ C_r log \frac{\sigma_p}{\sigma_{vo}} + C_c log \frac{(\sigma_{vo} + \Delta \sigma_p)}{\sigma_p} \}$

$S_c = \frac{2}{1 + 1.03} \left\{ 0.05 log \frac{204.6}{136.4} + 0.3 log \frac{(276.4)}{204.6} \right\} = 0.047 \text{ m} \quad 47\text{mm}$

**Secondary Compression Settlement $S_s$**

Secondary compression settlement takes place at constant effective stress, when there is no more dissipation of excess pore water pressure. For simplicity, it is assumed to start occurring when the primary consolidation is completed at time $t_p$, and the settlement increases linearly with the logarithm of time. Secondary compression settlement can be estimated using the following equation:

$$S_s = \frac{H}{1 + e_p} C_s log \frac{t}{t_p} \quad t > t_p$$

$C_s$ : Secondary Compression Index; Is the slope of the third part of consolidation curve in a plot of the logarithm of vertical effective stress verses void ratio.
Settlement of Shallow Foundations in Granular Soils

Settlement of footings in granular soils is instantaneous, with some possibility for long-term creep. The five most important factors that govern the settlement of a footing are the applied pressure, soil stiffness, footing breadth, footing depth, and footing shape. Soil stiffness often is quantified indirectly through penetration resistance such as the N-value or blow count from a standard penetration test or through tip resistance qc from a cone penetration test.

Terzaghi and Peck Method

Terzaghi and Peck (1967) proposed the first rational method for predicting settlement of a shallow foundation in granular soils. They related the settlement of a square footing of width B (in meters) to that of a 300-mm square plate, obtained from a plate loading test, through the following expression:

\[ \delta_{footing} = \delta_{plate}\left(\frac{2B}{B + 0.3}\right)^2 \left(1 - \frac{1}{4} \frac{D_f}{B}\right) \]

The last term in the equation accounts for the reduction in settlement with the increase in footing depth. Leonards (1986) suggested replacing 1/4 by 1/3, based on additional load test data. The values of \( \delta_{plate} \) can be obtained from Figure below, which summarizes the plate loading test data given by Terzaghi and Peck (1967). This method originally was proposed for square footings, but can be applied to rectangular and strip footings with caution. The deeper influence zone and increase in the stresses within the soil mass in the case of rectangular or strip footings are compensated for by the increase in the soil stiffness.

![Graph showing settlement of 300-mm x 300-mm plate](image)
6. Retaining Walls

A retaining wall is a structure whose primary purpose is to provide lateral support to soil and rock. Some of the common types of retaining walls are:

a) Gravity retaining Walls.
b) Semi-Gravity Retaining Walls.
c) Cantilever Retaining Walls.
d) Counterfort Retaining Walls.

Common Proportion of Cantilever Concrete Retaining Wall

A cantilever retaining wall is built of reinforced concrete. It consists of a thin stem and a base slab. The stem of a cantilever retaining wall is provided with reinforcement at the back. It also is provided with temperature reinforcement near the exposed front face to control cracking that might occur due to temperature changes. Dimensions for the concrete retaining walls should be adequate for stability and structural requirements, so tentative dimensions can be used and then revised accordingly.

Stability Checks of a Retaining Wall

The following stability checks are necessary for a retaining wall.
1. Stem shear and bending due to lateral earth pressure on the stem. This is a separate analysis using the stem height.
2. Base shear and bending moments at the stem caused by the wall loads producing bearing pressure beneath the wall footing (or base). The critical section for shear should be at the stem faces for both toe and heel. Toe bending is seldom a concern but for heel bending the critical section should be taken at the approximate center of the stem reinforcement and not at the stem back face.
(a) Wall pressure to use for shear and bending moment in stem design. Also shown is bearing capacity pressure diagram using $B' = b - 2e$ and $L = L' = 1$ unit.

(b) Wall pressure for overall stability against overturning and sliding. $W_c =$ weight of all concrete (stem and base); $W_s =$ weight of soil in zone acde. Find moment arms $x_i$ any way practical — usually using parts of known geometry. Use this lateral pressure for base design and bearing capacity.

1. Overturning Stability (Rotation about the Toe)

The factor of safety $F_{OT}$ against overturning of a wall about its toe (point O) can be determined using

$$F_{OT} = \frac{\text{Resisting Moment}}{\text{Overturning Moment}} = \frac{\sum W_i \times x_i + P_{aw} \times B + P_p \times \bar{y}_p}{P_{ah} \times \bar{y}_a}$$

$\geq 1.5$ for cohesionless soil, $2.0$ for cohesive soil
2. Sliding Stability (Translation)
The sliding stability along the base of the wall needs to be checked. The factor of safety for sliding stability is calculated as;

\[ F_s = \frac{\text{Resisting Forces}}{\text{Driving Forces}} = \frac{\sum V \tan \delta + C_a BL + P_p}{P_{ah}} \]

\[ \geq 1.5 \text{ for cohesionless soil, } 2.0 \text{ for cohesive soil} \]

\[ \delta = 0.67 \phi \]
\[ C_a = 0.6C \]

Base keys may be located at different positions under the base to increase the resisting forces in case adequate factor of safety against sliding is not achieved.

The most effective location of the base key which increases the length of the slip line.
3. Bearing Capacity Stability

Retaining walls should have adequate factor of safety against bearing capacity failure, ultimate bearing capacity may be determined using Hansen’s bearing capacity equations to include the effects of eccentric and inclined loading on the base, with $\alpha_1=2$, $\alpha_2=3$ and $s_i=1$ for strip footing

$$q_{ult} = c N_c d_c i_c + \alpha q N_q d_q i_q + 0.5 B' \gamma N_Y d_Y i_y$$

$$q_{ult} = \frac{q_{ult}}{F_B}$$

$F_B=2.0$ for cohesionless soil, $3.0$ for cohesive soil

The actual bearing pressure under the base of the wall can be determined by

$$\sigma_{min,max} = \frac{P}{A} \pm \frac{M c}{I}$$

$$q_{min,max} = \frac{R}{B} \pm \frac{R e B/2}{B^3/12} = \frac{R}{B} \left(1 \pm \frac{6 e}{B}\right) \text{for unit length of wall}$$

Provided that:
1. $q_{max} \leq q_{ult}$
2. $e \leq B/6$ so that $q_{min}$ not be negative

Example: Analyze the retaining wall for overall stability?

![Diagram of retaining wall with dimensions and loads](image)
Solution:

Rankine $K_a = 1 - \sin28/1+\sin28 = 0.36$

At top
$\sigma_a = 23.9 \times 0.36 - 2 \times 19.12 \times \sqrt{0.36} = -14.4 \text{ kPa}$

At base
$\sigma_a = (23.9 + 17.95 \times 6.7) \times 0.36 - 2 \times 19.12 \times \sqrt{0.36} = 32 \text{ kPa}$

$P_a = 4.62 \times 32/2 \text{ soil } + \text{ water } 10 \times 2.1 \times 2.1/2 = 96 \text{ kN}$

$Y'$ from base = 4.62/3 = 1.54 m $Y'$ from base = 6 m

Assume $P_a$ acts horizontally because of small inclination angle of the wall

<table>
<thead>
<tr>
<th>Part</th>
<th>Weight</th>
<th>kN</th>
<th>Arm m</th>
<th>Moment kN.m</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1.5x23.9+1.5 x6.1x17.95</td>
<td>200</td>
<td>2.13</td>
<td>426</td>
</tr>
<tr>
<td>2</td>
<td>0.25x6.1x23.56</td>
<td>36</td>
<td>1.125</td>
<td>40.5</td>
</tr>
<tr>
<td>3</td>
<td>0.13x(23.9+6.1x(23.56+17.95)/2)</td>
<td>19.55</td>
<td>1.315</td>
<td>25.7</td>
</tr>
<tr>
<td>4</td>
<td>2.88x 0.6 x23.56</td>
<td>40.7</td>
<td>1.44</td>
<td>58.6</td>
</tr>
<tr>
<td>5</td>
<td>$\Sigma$</td>
<td>296.25</td>
<td></td>
<td>550.8</td>
</tr>
</tbody>
</table>

$P_{av} = P_{ah} \times \tan \delta = 96 \times \tan(2 \times 28/3)$

$F_{0,T} = \frac{\text{Resisting Moment}}{\text{Overturning Moment}} = \frac{550.8 + 93.4}{74 \times 1.54 + 22 \times 6} = 2.61 > 2 \text{ ok}$

$F_s = \frac{\text{Resisting Forces}}{\text{Driving Forces}} = \frac{\Sigma V \tan \delta + C_a BL + P_p}{P_{ah}}$

$= \frac{296.25 \times \tan \left(\frac{2 \times 34}{3}\right) + \left(\frac{2 \times 35.17}{3}\right) \times 2.88}{96} = 2 \text{ ok}$

$X = \frac{550.8 - (74 \times 1.54 + 22 \times 6)}{296.25} = 1.03 \text{ m from toe}$

$= 0.41 \text{ m from center} < \left(\frac{2.88}{6} = 0.48\right)$

Means that; the base subjected to a concentrated force of 296.25 at center and counterclockwise moment of 296.25 x 0.41 = 121.5 kNm.

$q_{min, max} = \frac{R}{B} \left(1 \div \frac{6 e}{B}\right) = \frac{296.25}{2.88} \left(1 \div \frac{6 \times 0.41}{2.88}\right) = 15 \text{ kPa, 190.7 kPa}$
Stability of Flexible Retaining Walls (sheet Pile)

Types and Materials Used for Sheetpiling
Sheetpiling materials may be of timber, reinforced concrete, or steel, steel sheetpiling is the most common type used for walls because of several advantages over other materials
1. It is resistant to the high driving stresses developed in hard or rocky material.
2. It is relatively lightweight.
3. It may be reused several times.
4. It has a long service life either above or below water.
5. It is easy to increase the pile length by either welding or bolting.
6. Joints are less apt to deform when wedged full with soil and small stones during driving.
7. A nearly impervious wall can be constructed.

The common type of sheet pile retaining walls are;
1. Cantilever wall commonly used to support soil of a height of less than three meters.
2. Anchored or propped sheet pile wall commonly used to support deep excavation and waterfront retaining structure.

In analyzing sheet pile walls, we are attempting to determine the depth of embedment, $d$, for stability. The analysis is not exact and various simplifications are made. The key static equilibrium condition is moment equilibrium. Once we determine $d$, the next step is to determine the size of the wall. This is done by calculating the maximum bending moment and then determining the section modulus by dividing the maximum bending moment by the allowable bending stress of the material constituting the sheet pile, for example, steel, concrete, or wood.

![Approximation of Pressure Distribution in the Analysis of Cantilever Flexible Retaining Walls](image-url)
Example:

Determine the depth of embedment required for stability of the cantilever sheet pile wall shown in figure.

Solution:

1. Find depth of embedment

Determine the lateral earth pressure distribution

\[ P_a = \frac{1}{2} K_a \gamma (H_o + d_o)^2 \]

\[ Mo_a = P_a \times (H_o + d_o)/3 \]

\[ Mo_a = \frac{1}{2} K_a \times 18 \times (H_o + d_o)^3 /3 = 3 K_a (3+d_o)^3 \]

\[ P_p = \frac{1}{2} K_p \gamma (d_o)^2 \]

\[ Mo_p = \frac{1}{2} K_p \times 18 \times (d_o)^3 /3 = 3 K_p (d_o)^3 \]

By Factored Moment Method (FMM) use \( \phi \) to determine \( K_a \) and \( K_p \) and factor of safety \( (FS)_p = 1.5 - 2 \) to overcome the rotation of sheet pile around point \( o \) due to the active earth pressure.

\[ K_a = 1/3 \quad \text{and} \quad K_p = 3 \]

\[ (FS)_p = 1.5 = \frac{Mo_p}{Mo_a} \]

\[ \frac{(3+d_o)^3}{d_o^3} = 6 \quad \text{by trial and error} \quad d_o = 3.65 \text{ m} \]

By Factored Strength Method (FSM) factor of safety \( F_\phi = 1.2 - 1.5 \) is applied to the friction angle to determine the active and passive earth pressure from \( \phi_{design} = \phi / F_\phi \).

\[ \phi_{design} = \phi / F_\phi = 30^\circ / 1.25 = 24^\circ \]

\[ K_a = 0.42 \quad \text{and} \quad K_p = 2.38 \]

\[ Mo_a = 3 K_a (3+d_o)^3 = 3 \times 0.42 \times (3+d_o)^3 = 1.26 (3+d_o)^3 \]

\[ Mo_p = 3 K_p \times (d_o)^3 = 3 \times 2.38 \times (d_o)^3 = 7.14 (d_o)^3 \]

\[ Mo_a = Mo_p \Rightarrow 1.26 (3+d_o)^3 = 7.14 (d_o)^3 \]

\[ \frac{(3+d_o)^3}{d_o^3} = 5.66 \quad \text{by trial and error} \quad d_o = 3.85 \text{ m} \]
By Net Passive Pressure Method (NPPM) use $\phi$ to determine $K_a$ and $K_p$ and factor of safety $(FS)_r = 1.5 - 2$ to overcome the rotation of sheet pile around point $O$ due to the active earth pressure distribution as in figure shown.

$K_a = 1/3$ and $K_p = 3$

$P_{a1} = \frac{1}{2} K_a \gamma (H_0)^2 = \frac{1}{2} \times \frac{1}{3} \times 18 \times 3^2 = 27 \text{kN/m}$

$P_{a2} = K_a \gamma H_0 d_0 = \frac{1}{3} \times 18 \times 3 \times d_0 = 18 \text{d}_0 \text{kN/m}$

$M_{o_a} = 27 (d_0 + 1) + 18d_0 \times d_0/2 = 9d_0^2 + 27d_0 + 27$

$P_p = \frac{1}{2} (K_p - K_a) \gamma (d_0)^2 = \frac{1}{2} (3 - 1/3) 18 \text{d}_0^2 = 24 \text{d}_0^2$

$M_{o_p} = 24 \text{d}_0^2 \times d_0/3 = 8 \text{d}_0^3$

$\frac{(FS)_r}{(FS)_r = 1.5} = \frac{M_{o_p}}{M_{o_a}}$

$5.33d_0^3 - 9d_0^2 - 27d_0 = 27$ by trial and error $d_0 = 2.8 \text{m}$

The design depth $d = (1.2 - 1.3) d_0$

FMM $d = 1.2 \times 3.65 = 4.38 \text{m}$

FSM $d = 1.2 \times 3.85 = 4.62 \text{m}$

NPPM $d = 1.2 \times 2.8 = 3.36 \text{m}$

2. Find force $R$

FMM $R = P_p - P_a = \frac{1}{2} K_p \gamma (d_0)^2 - \frac{1}{2} K_a \gamma (H_0+d_0)^2 = 1/2 \times 3 \times 18 \times 3.65^2 - \frac{1}{2} \times 1/3 \times 18 \times (3+3.65)^2 = 227 \text{kN}$
To calculate the net resistance below the assumed point of rotation, O, calculate the average passive pressure at the back of the wall and the active pressure in front of the wall. Notice that below the point of rotation, passive pressure acts in front of the wall and active pressure acts at the back of the wall.

Find the pressure at 1.1 d₀, which is the average pressure at the extra (0.2 d₀) length;

Average Active lateral pressure = \( \frac{K_a}{\gamma} \times 1.1d_0 = \frac{1}{3} \times 18 \times 1.1 \times 3.65 = 24.09 \) kPa

Average passive lateral pressure = \( \frac{K_p}{\gamma} (H_0+1.1d_0) = 3 \times 18 \times (3 + 1.1 \times 3.65) = 378.81 \) kPa

Net force below the rotation point O = (378.81-24.09) x 0.2 x 3.65 = 258.94 kN > R

3. Find the size of the section, Determine the maximum moment

\[ M_z = \frac{1}{2} K_a \gamma \left( \frac{z}{3} \right)^2 - \frac{1}{2} K_p \gamma (z-3)^2 (z-3)/3 \]

dM/dz = 0 =

\[ M_{max} = \]

\[ M_{max}/\sigma_{all} = I/c \text{ section modulus} \]

Example:

Determine the embedment depth and the anchor force of the tied-back wall shown in figure, using FSM?

Solution:

Use Coulomb general equation to find \( K_{ax} \) and \( K_{px} \)

\[ \phi_{design} = \frac{\phi}{F_\phi} = 30°/1.2 = 25° \]

\( K_{ax} = 0.42 \)

\( K_{px} = 3.4 \)
1. Find $d_0$: Assume $R$ at base = 0

$$M_t)_a = 0.42 \times 10 \times (8 + d_0)(8/2 + d_0/2 - 1) + 1/2 \times 0.42 \times 18 \times 8^2 \times (2 \times 8/3 - 1) + 0.42 \times 18 \times 8 \times d_0 \times (7 + d_0/2) + 1/2 \times 0.42 \times 10 \times d_0^2 \times (7 + 2d_0/3)$$

$$M_t)_p = 1/2 \times 3.4 \times 10 \times d_0^2 \times (7 + 2d_0/3)$$

$$M_t)_a = M_t)_p \quad d_0 = 5.4m$$

2. Find T force at anchor

$$\sum F_x = 0$$

$$T =$$
7. Pile Foundation (Deep Foundation)

Pile; a slender structural element consisting of steel or concrete or timber that transfers the load at deeper depths is called a deep foundation. Selection of the type of foundation generally is based on many factors, including but not limited to the magnitude and type of the design load, strength and compressibility of site soils, project performance criteria, availability of foundation construction materials, and foundation cost.

End bearing or point bearing pile is one that transfers almost all the structural load to the soil at the bottom end of the pile.

Friction pile is one that transfers almost all the structural load to the soil by skin friction along a substantial length of the pile.

Floating pile is a friction pile in which the end bearing resistance is neglected.

Why Pile Foundation?
1. The soil under surface does not have sufficient bearing capacity to support the structural loads.
2. The estimated settlement of the soil exceeds tolerable limits (settlement greater than the serviceability limit state).
3. Differential settlement due to soil variability or non-uniform structural loads is excessive
4. The structural loads consist of lateral loads and/or uplift forces.
5. Excavations to construct a shallow foundation on a firm soil layer are difficult or expensive.

Types of Piles Materials and Installation

Concrete piles
Several types of concrete piles are commonly used; these include cast-in-place concrete piles, precast concrete piles. Cast—in-place concrete piles are formed by driven a cylindrical steel shell into the ground to the desired length and then filling the cavity of the shell by fluid concrete. Precast concrete piles usually have square or circular or octagonal cross section and are fabricated in construction yard from reinforced or prestressed concrete.

Steel Piles
Steel pile come in various shapes and sizes and include cylindrical seamless pipe, tapered and H—piles which is rolled steel sections, concrete-filled steel pile can be done by replacing the soil inside the tube by concrete to increase the load capacity.

Timber Piles
Timber piles have been used since ancient times with a common length about 12 meters.

Pile Installation
Piles can either be driven into the ground (driven piles) or be installed in a predrilled hole (bored piles or drilled shafts).
1. Driving with a steady succession of blows on the top of the pile using a pile hammer. This produces both considerable noise and local vibrations, which may be disallowed by local codes or environmental agencies and, of course, may damage adjacent property.
2. Driving using a vibratory device attached to the top of the pile. This method is usually relatively quiet, and driving vibrations may not be excessive. The Method is more applicable in deposits with little cohesion.

3. Jacking the pile. This technique is more applicable for short stiff members.

4. Drilling a hole and either inserting a pile into it or, more commonly, filling the cavity with concrete, which produces a pile upon hardening.

**Axial Capacity of Piles in Compression**

Axial capacity of piles primarily depends on how and where the applied loads are transferred into the ground. Based on the location of the load transfer in deep foundations, they can be classified as follows:

1. **End- or point-bearing piles**: The load is primarily distributed at the tip or base of the pile.

2. **Frictional piles**: The load is distributed primarily along the length of the pile through friction between the pile material and the surrounding soil.

3. **Combination of friction and end bearing**: The load is distributed both through friction along the length of the pile and at the tip or base of the pile.

\[
P_{ult} = P_p + P_s
\]

**Pile in Cohesionless Soil**

1. **Point Capacity**

If we incorporate the effect of shape and depth in determination of the N factors, the equation for bearing capacity of shallow foundations may be modified for deep foundations after neglecting the third part because of the small diameter or width of the piles as:

\[
q_{ult} = c N'_c + q N'_q
\]

\[
P_{pu} = (c N'_c + q N'_q) A_p
\]

**Figure 5.16** Meyerhof (1976) bearing capacity factors \( N'_c \) and \( N'_q \) (adapted from Das 1999).
Meyerhof Method  Cohesionless soil

\[ P_{pu} = \bar{q}N_q^* A_p < (50 N_q^* \tan \phi) A_p \text{ kN} \]

2. Skin friction Capacity

Field studies have shown that the unit frictional resistance of piles embedded in cohesionless soils increases with depth. However, beyond a certain depth, the unit frictional resistance remains more or less constant, as illustrated; this depth, beyond which the unit frictional resistance does not increase, is called the critical depth and has been observed to vary between 15 to 20 times the pile diameter.

\[ P_s = \sum A_s f_s \]

where

- \( A_s \) = effective pile surface area on which \( f_s \) acts
- Skin resistance \( f_s = K \sigma_v' \tan \delta \)
- \( K = K_0 \) Bored or jetted piles
- \( K = 1.4K_0 \) Low-displacement driven piles
- \( K = 1.8K_0 \) High-displacement driven piles

where \( K_0 = 1 - \sin \phi \) for sands.

Example:

A concrete pile is 15 m (L) long and 0.4x0.4 m in cross section, the pile is fully embedded in sand for which \( \gamma = 15.5 \text{ kN/m}^3 \), and \( \phi=30^\circ \). Calculate;

1. The ultimate point load of the pile?
2. The frictional resistance force if \( K=1.3 \) and friction angle between pile and soil \( \delta=0.8\phi \)?
3. The allowable pile load, \( FS=4 \)?

Solution:

1. Using Meyerhof Method

\[ P_{pu} = \bar{q}N_q^* A_p \]

From figure, for \( \phi = 30^\circ \), \( N_q^* = 55 \)

\[ P_{pu} = 15.5 \times 15 \times 55 (0.4x0.4) = 3046 \text{ kN} \]

Check with max. limit \( (50 N_q^* \tan \phi) A_p = 50 \times 55 \times 0.577 \times 0.4x0.4 = 254 \text{ kN} \)

Use \( P_{pu} = 254 \text{ kN} \).
Using Hansen's equation?

2.

\[ F_s) = K \sigma' \tan \delta = 0 \]

Critical depth = 20x pie diameter = 20x0.4 = 8m

\[ F_s) = K \sigma' \tan \delta = 1.3 \times 15.5 \times 8 \times \tan (0.8 \times 30) = 71.7 \text{ kN/m}^2 \]

\[ P_s = \frac{0 + 71.7}{2} \times (4 \times 0.4 \times 8) + 71.7 \times (1.6 \times 7) = 1262 \text{ kN} \]

3.

\[ P_{ult} = P_p + P_s = 254 + 1262 = 1516 \text{ kN} \]

\[ P_{all} = \frac{P_{ult}}{F_s} = 1516 / 4 = 379 \text{ kN} \]

Pile in Cohesive Soil

1. Point Capacity

In clay \( \phi = 0 \)

\[ q_{ult} = c N_c^* \]

Bearing capacity factor \( N_c^* \) is commonly taken as 9

\[ P_{pu} = 9 c A_p \]

2. Skin friction Capacity

\[ P_s = \sum A_s f_s \]

\( f_s = \alpha c \)

Where

\( \alpha \) = coefficient from figure

\( c \) = average cohesion (or\( S_u \)) for the soil stratum of interest

![Graph showing \( \alpha \)-factor vs. \( S_u \)](image)
Example:
A driven-pipe pile in clay is shown in figure. The pipe has an outside diameter of 406 mm and a wall thickness of 6.35 mm.

a. Calculate the net point bearing capacity.
b. Calculate the skin resistance.
c. Estimate the net allowable pile capacity.

Use FS = 4.

Solution:

a. 
\[ P_{pu} = 9 \cdot 100 \cdot (3.14 \cdot 0.406^2 / 4) = 116.5 \text{ kN} \]

b. perimeter of the pile = 0.406 \times 3.14 = 1.275 \text{ m}

\[ P_s = 30 \times 0.95 \times 5 \times 1.275 + 30 \times 0.95 \times 5 \times 1.275 + 100 \times 0.72 \times 20 \times 1.275 = 2200 \text{ kN} \]

c. 
\[ P_{ult} = P_{p} + P_{s} = 116.5 + 2200 = 2316.5 \text{ kN} \]

\[ P_{all} = P_{ult} / FS = 2316.5 / 4 = 580 \text{ kN} \]

Using Penetration Test Data (SPT) for Pile Capacity

Meyerhof suggested that for piles embedded at least 10 pile diameters in the sand or gravel-bearing stratum, the point capacity can be approximated using SPT data as

\[ P_{pu} = A_p (40N) \frac{L_b}{B} \leq A_p (380N) \text{ kN} \]

Where

\[ N = \text{statistical average of the SPT N}_{55} \text{ numbers in a zone of about } 5B \text{ above to } 3B \text{ below the pile point} \]

\[ B = \text{width or diameter of pile point} \]

\[ L_b = \text{pile penetration depth into point-bearing stratum} \]

\[ L_b / B = \text{average depth ratio of point into point-bearing stratum} \]

Meyerhof also suggested that skin friction capacity of piles embedded in sand or gravel can be approximated using SPT data as follows.

\[ P_s = 2(N_{55}) A_s \leq 100 \ A_s \]

According to Shioi and Fukui (1982) pile tip resistance is computed in Japan as

\[ P_{pu} = q_{ult} A_p \]
With the ultimate tip bearing pressure $q_{ult}$ computed from the SPT based on the embedment depth ratio $L_b/D$ into the point-bearing stratum as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Formula</th>
</tr>
</thead>
</table>
| Driven piles  | $q_{ult} = N \left(6L_b/D\right) \leq 30$ for Open-end pipe pile  
                | $q_{ult} = N \left(10 + 4L_b/D\right) \leq 30$ for Closed-end pipe pile |
| Cast-in-place | $q_{ult} = 300$ in Sand                 
                | $q_{ult} = 3S_u$ in Clay               |
| Bored piles   | $q_{ult} = 10N$ in Sand                
                | $q_{ult} = 15N$ in Gravelly Sand       |

Where this SPT $N$ should be taken as $N_{55}$.

**Using Cone Penetration Test data (CPT)**

With $L/B \geq 10$ the point load is estimated by the Japanese as

$$P_{pu} = \gamma_p q_o$$

Where $q_c = \text{statistical average of the cone point resistance in a zone similar to that for } N_{55}$

**Example:**

An HP360 X 132 (14 X 89) pile penetrates through 9 m of soft clay and soft silty clay, into 1 m of a very dense gravelly sand for a total pile length $L = 10$ m. The GWT is at 1.5 m below the ground surface. The pile was driven essentially to refusal in the dense sand. The SPT $N_{70}$ blow count prior to driving ranged from 3 to 10 in the soft upper materials and from 40 to 60 in the dense sand.

On this basis it is decided to assume the pile is point bearing and receives no skin resistance contribution from the soft clay. We know there will be a considerable skin resistance contribution, but this design method is common. We will make a design using Meyerhof's Eq.

**Solution:**

If we use an average blow count $N_{70} = 50$,

$$N_{55} = N_{70} \times \frac{70}{55} = 50 \times \frac{70}{55} = 64$$
The projected point area of the pile section is $A_p = 0.351 \times 0.373 = 0.131 \text{ m}^2$

$L_B/B = 1 / 0.351 = 2.85$

$P_{pu} = A_p \left( 40N \right) \frac{L_B}{B} = 0.131 \times (40 \times 64) \times 2.85 = 956 \text{ kN}$

Check maximum limit $P_{pu} = A_p \left( 380 \text{ N} \right) = 0.131 \times 380 \times 64 = 3186 \text{ kN} \quad \text{OK Use 956 kN}$

**Pile Load Test**

The purposes of a pile load test are:

- To determine the axial load capacity of a single pile.
- To determine the settlement of a single pile at working load.
- To verify the estimated axial load capacity.
- To obtain information on load transfer in skin friction and end bearing.

The allowable bearing capacity is found by dividing the ultimate load, found from the load settlement curve, by a factor of safety, usually 2. An alternative criterion is to determine the allowable pile load capacity for a desired serviceability limit state, for example, a settlement of 10% of the pile diameter. Also pile settlement under double working load should not be more than 25 mm.
EFFICIENCY OF PILE GROUPS

When several pile butts are attached to a common structural element termed a pile cap the result is a pile group. A question of some concern is whether the pile group capacity is the sum of the individual pile capacities or something different—either more or less. If the capacity is the sum of the several individual pile contributions, the group efficiency \( E_g = 1.0 \).

Optimum spacing \( s \) seems to be on the order of 2.5 to 3.5D or 2 to 3H for vertical loads where \( D = \) pile diameter; \( H = \) diagonal of rectangular shape or HP pile. Group efficiency can be estimated using

\[
E_g = 1 - \frac{(n - 1)m + (m - 1)n}{90mn}
\]

Where \( m, n \) are no. of columns and rows of piles

\( \theta = \tan^{-1} \frac{D}{s} \) in degrees

**Function of Pile Cap**

1. Transfer column load to pile bed.
2. To substitute the ill effect of one pile to others
3. To take any deviation in the location of piles

**Minimum Total Thickness of Pile Cap**

150 mm pile penetration in cap
75 mm concrete cover for cap steel above pile
Twice bar diameter
300 mm minimum concrete thickness above reinforcement
Example:
Design the pile cap for the case shown
Footing size = 2.6 × 2.6 m.
Column size = 0.4 × 0.4 m.
Pile diameter = 0.3 m. c.c = 0.9 m
\( f_c' = 30 \text{ MPa} \)
Load per pile:
\( P_0 = 90 \text{ kN}, \quad P_L = 45 \text{ kN} \)

Solution
\[
E_g = 1 - \frac{\theta (n - 1)m + (m - 1)n}{90mn}
\]
\[
\theta = \tan^{-1} \left( \frac{0.3}{0.9} \right) = 18.43^\circ
\]
\[
E_g = 1 - 18.43 \times \frac{3(3) + 3(1)}{90 \times 3 \times 3} = 0.72
\]

Ultimate Pile Load
\( P_u = 1.4 \times 90 + 1.7 \times 45 = 202.5 \text{ kN} \)

Find Depth of Footing Using Shear Strength
1. Wide Beam Shear – Section at \( d \) from column face
\[
V_u = 3 \times 202.5 = 607.5 \text{ kN}
\]
\[
\phi V_c = 0.85 \times 0.17 \sqrt{f_c'} b_w d = 0.85 \times 0.17 \sqrt{30} \times 2.6 \times d \times 1000 = 2057 \text{ d}
\]
\[
d = \frac{607.5}{2057} = 0.3 \text{ m}
\]
2. Two-Way Shear – Section at \( d/2 \) from column face
\[
V_u = 8 \times 202.5 = 1620 \text{ kN}
\]
\[
\phi V_c = 0.85 \times 0.33 \sqrt{f_c'} b_o d = 0.85 \times 0.33 \times \sqrt{30} \times 4(0.4 + d) d \times 1000 = 1536 (1.6d + 4d^2)
\]
\[
1620 = 1536 (1.6d + 4d^2) \quad \text{d} = 0.35 \text{ m}
\]
3. Check Punching Shear Strength at Corner Column.
\[
P_u = 202.5 \text{ kN}
\]
\[
\phi V_c = 0.85 \times 0.33 \sqrt{f_c'} b_o d
\]
\[
= 0.85 \times 0.33 \times \sqrt{30} \times 3.14 (0.3 + d) d \times 1000 = 4824 (0.3d + d^2)
\]
\[
d = 0.1 \text{ m}
\]

Use \( d = 350 \text{ mm} \)
8. DESIGN OF SHALLOW FOUNDATION

Foundation Design Requirements

1. Shear stresses transmitted to the soil must be smaller than the shearing resistance of soil by an amount which gives an ample factor of safety.
2. The settlement “especially differential settlement” must not reach a degree which might damage the structure.
3. Footing must be placed at adequate depth to prevent frost damage... etc.

Footing Depth and Spacing

Footings should be constructed below

1. The frost line
2. Zones of high volume change due to moisture fluctuations
3. Top soil or organic material
4. Peat and muck
5. Unconsolidated material such as abandoned (or closed) garbage dumps and similar filled-in areas.

Underground defects or utilities may affect the foundation depth, for example, limestone caverns, soft material, sewer tunnels, telephone-cable conduits,

Presence of Adjacent Footing

When footings are to be placed adjacent to an existing structure, the line from the base of the new footing to the bottom edge of the existing footing should be 45° or less with the horizontal plane. $M > Z_f$

Conversely, Fig. b indicates that if the new footing is lower than the existing footing, there is a possibility that the soil may flow laterally from beneath the existing footing. This may increase the amount of excavation somewhat but, more importantly, may result in settlement cracks in the existing building. This problem is difficult to analyze; however, an approximation of the safe depth $Z_f$ may be made for a $\phi$-C soil. By equating $\sigma_3 = 0$ on the vertical face of the excavation;

$$\sigma_1 = \gamma Z_f + q_0$$
$$\sigma_3 = \sigma_1 K - 2c\sqrt{K} = \gamma Z_f K + q_0 K - 2c \sqrt{K} = 0$$

$$Z_f = \frac{2c}{\gamma \sqrt{K} - q_0 / \gamma}$$

Using factor of safety $F$

$$Z_f = \frac{2c/F \gamma \sqrt{K} - q_0 / \gamma}{\gamma}$$

(a) An approximation for the spacing of footings to avoid interference between old and new footings. If the “new” footing is in the relative position of the “existing” footing of this figure, interchange the words “existing” and “new.” Make $m > z_f$.

(b) Possible settlement of “existing” footing because of loss of lateral support of soil wedge beneath existing footing.
Allowable Soil Pressure

Allowable soil pressure ($q_a$) can be determined using bearing capacity equations ($q_{ult}$) with a proper factor of safety depending on the type of soil, loading conditions and foundation geometry.

**NOTICE:** Alternatively, when SPT samples are used to find $q_u$ (Unconfined compressive strength of cohesive soil, $\phi=0$) a nearly universal value is used of allowable bearing capacity ($q_a = q_u$) for square footing of unknown dimension and uncertain depth using safety factor SF=3.0.

**Net pressure:** pressure in excess of the existing overburden pressure that can be safely carried at the foundation depth D (based on settlement limitations)?

**Gross pressure:** the total pressure that can be carried at the foundation depth, including the existing overburden pressure (and based on soil strength considerations)?

The bearing-capacity equations are based on gross soil pressure $q_{ult}$ which is everything above the foundation level. Settlements are caused only by net increases in pressure over the existing overburden pressure.

Soil Pressure Assumption

The function of footing is to "spread" the column load laterally to the soil so that the stress intensity is reduced to a value that the soil can safely carry. The pressure distribution beneath most footings will be rather indeterminate because of the interaction of the footing rigidity with the soil type, state, and time response to stress. For this reason it is common practice to use the linear pressure distribution beneath spread footings. The few field measurements reported indicate this assumption is adequate.

Spread Footing

**Spread footing:** A footing carrying a single column, wall footings serve a similar purpose of spreading the wall load to the soil.
1. Square Footing

Example:

Design a plain (unreinforced) concrete spread footing for the following data:

DL = 90 kN  LL= 100 kN  
Column: W 200 X 31.3 resting on a 220 X 180 X 18 mm base plate

\( f_{c} = 21 \text{ MPa} \)

Allowable net soil pressure \( qa = 200 \text{ kPa} \)

**ACI 318 Relevant clauses**

22.4.8 — When computing strength in flexure, combined Flexure and axial load, and shear, the entire cross section of a member shall be considered in design, except for concrete cast against soil where overall thickness \( h \) shall be taken as 50 mm less than actual thickness.

22.7.4 — Thickness of structural plain concrete footings shall be not less than 200 mm. See 22.4.8.

15.4.2 — Maximum factored moment, \( M_u \), for an isolated footing shall be computed as prescribed in 15.4.1 at critical sections located as follows:

(a) At face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall;
(b) Halfway between middle and edge of wall, for footings supporting a masonry wall;
(c) Halfway between face of column and edge of steel base plate, for footings supporting a column with steel base plate.

C.3.5 — In Chapter 22, \( \phi \) shall be 0.65 for flexure, compression, shear, and bearing of structural plain concrete

**Solution:**

1. **Find Size of Footing**

\[
A = \frac{P}{qa} = \frac{(90+100)}{200} = 0.95 \text{ m}^2
\]

\( B = \sqrt{A} = \sqrt{0.95} = 0.97 \text{ m} \)  Use 1 m

2. **Find Thickness of Footing**

\[
P_{ult} = 1.4DL + 1.7LL = 1.4(90) + 1.7(100) = 296 \text{ kN}
\]

\[
q_{ult} = \frac{P_{ult}}{A} = \frac{296}{1} = 296 \text{ kPa}
\]

\[
f_t = 0.42\phi\sqrt{f_{c}'} = 0.42 \times 0.65 \times \sqrt{21} = 1.19 \text{ MPa}
\]

**ACI 318 - R22.7.4**
M\text{u}= \frac{w l^2}{2} = 296 \times (0.5 - 0.18/4)^2 /2 = 30.64 \text{ kN.m/m} \quad \text{ACI 318 - 15.4.2 - c}

f_t = M/S , S = bd^2/6 \quad d = \sqrt[3]{30.64\times6/(1.19\times1\times1000)} = 0.39 \text{ m}

Overall thickness \( h = d + 50 \text{ mm} = 0.39 + 0.05 = 0.44 \text{ m} \quad \text{Use 0.45m} \quad \text{ACI 318 - 22.4.8}

**Check Two Way Shear Action**

\[ V_u \leq \psi V_c \]

\[ V_c = 0.11(1 + 2/\beta) \sqrt{f'_c \cdot b_o \cdot d} \leq 0.22 \sqrt{f'_c \cdot b_o \cdot d} \quad \text{ACI 318 – 22.5.4} \]

Using \( d = 0.4 \text{ m} \) and effective column dimensions \( 210 \times 134 \text{ mm} \)

\[ b_o = 2(0.215 + 0.4 + 0.157 + 0.4) = 2.34 \text{ m} \]

\[ V_c = 0.11(1 + 2/ (210/134)) \sqrt{21 \times 2.34 \times 0.4} = 1074 \text{ kN} \leq 0.22 \sqrt{21 \times 2.34 \times 0.4} = 943 \text{ kN} \]

296 more less than \((0.65 \times 943) = 613 \text{ kN} \quad \text{ok} \]

**Check Beam Shear Action**

\[ V_u = 0.11 \sqrt{f'_c \cdot b_w \cdot d} = 0.11 \sqrt{21 \times 1 \times 0.4} = 201 \text{ kN} \]

\[ V_u = 296 \times 1 \times (0.5-0.4-0.157/2) = 6.37 \text{ kN} \]

0.65x201 >> 6.37 \quad \text{ok}

**Example:**

Determine the base area (A) required for a square spread footing with the following design conditions:

Service dead load = 1400 kN

Service live load = 1250 kN

Service surcharge = 5 kPa

Assume average weight of soil and concrete above footing base= 20 kN/m³

Depth of footing base = 1.5 m below floor level

Allowable soil pressure at bottom of footing = 200 kPa

Column dimensions = 0.75 x 0.3 m , \( f'_c = 25 \text{ MPa} \), \( f_y = 400 \text{ MPa} \)

**Critical Thinking:**

The 200 kPa means gross soil pressure even if it is not mentioned. The base area of the footing is determined using service (unfactored) loads with the net permissible soil pressure.

**Solution:**

**1. Determination of Base Area:**

\[ \text{ACI 318 – 15.2.2} \]

Weight of surcharge, soil and concrete above footing base = 5 + 20 x 1.5 = 35 kPa

Net allowable soil pressure = 200 - 35 = 165 kPa

2015 - 2016
Required base area of footing \( A = \frac{1400 + 1250}{165} = 16 \text{ m}^2 \)

Use a 4 x 4 m square footing

2. Factored Loads and Soil Reaction Are Used for Strength Purposes: \( \text{ACI 318 – 15.2.1} \)

\[
Pu = 1.4 \times 1400 + 1.7 \times 1250 = 4085 \text{ kN}
\]

\[ q_s = \frac{4085}{16} = 255 \text{ kPa} \]

**ACI 318 Relevant clauses**

11.1 — Shear strength

C.2.1 - \( \phi = 0.85 \)

11.1.1 - Design of cross sections subject to shear shall be based on:

\[
\phi V_n = V_u \quad (11-1)
\]

11.3.1.1 - For members subject to shear and flexure only,

\[
V_c = 0.17 \sqrt{f'_c b_w d} \quad (11-3)
\]

11.12.2 - The design of a slab or footing for two-way action

11.12.2.1 - For nonprestressed slabs and footings,
\( V_c \) shall be the smallest of (a), (b), and (c):

(a) \( V_c = 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f'_c \, b_o \, d} \quad (11-33) \)

where \( \beta \) is the ratio of long side to short side of the column, concentrated load or reaction area;

(b) \( V_c = 0.083 \left(a_s d/b_o + 2\right) \sqrt{f'_c \, b_o \, d} \quad (11-34) \)

Where \( a_s \) is 40 for interior columns, 30 for edge columns, 20 for corner columns; and

(c) \( V_c = 0.33 \sqrt{f'_c \, b_o \, d} \quad (11-35) \)

15.7 — Minimum footing depth

Depth of footing above bottom reinforcement shall not be less than 150 mm for footings on soil, nor less than 300 mm for footings on piles.

2. Find Footing Thickness

Check wide beam shear

\[
\phi V_c = 0.85 \times 0.17 \sqrt{f'_c \, b_w \, d} = 0.85 \times 0.17 \times \sqrt{25 \times 4 \times d \times 1000} = 2890 \text{ d}
\]

\[ V_u = 255 \times 4 \times (2 - d - 0.3/2) = 1020 (1.85 -d) \]

\[ V_u = \phi V_c \]

\[ 1887 - 1020d = 2890d \]

\[ d = 0.482 \text{ m} \]

Check two way shear \( \beta = 0.75/0.3 = 2.5 \)

\[
V_c / \sqrt{f'_c \, b_o \, d} = (a) = 0.17(1+2/2.5) = 0.306
\]

\[ (b) = 0.33 \quad \text{use 0.306} \]

\[
\phi V_c = 0.85 \times 0.306 \times \sqrt{f'_c \, b_o \, d} = 0.85 \times 0.306 \times 5 \times (2(0.75+0.3) + 4d) \times 1000
\]

\[ = 1300.5 (2.1d + 4d^2) \]


\[ V_u = 255 \left( 16 - (0.75 + d)(0.3 + d) \right) = 255 \left( 15.775 - 1.05d - d^2 \right) \]

\[ V_u = \phi V_c \]

\[ d = 0.626 \text{ m} > 0.15 \text{ m} \quad \text{ACI 318 - 15.7} \]

Use \( h = 0.626 + (0.1 \text{ cover} + \text{bar dia.}) \approx 0.75 \text{ m} \)

Check equation (11-34) for \( d = 0.626 \)

### 3. Find Flexural Reinforcement

\[ M_u = q_s l^2/2 = 255 \times (2 - 0.15)^2 /2 = 436.4 \text{ kN.m/m} \quad \text{ACI 318 - 15.4.2 - c} \]

\[ M_u = \phi A_s f_y (d - 0.5 A_s f_y / 0.85 f_c') \quad , = 0.9 \quad \text{ACI 318 - 9.3.2} \]

436.4 = 0.9 x \( A_s \) x 400 (0.6 - 0.5 \( A_s \) x 400 / 0.85x25)1000

\( A_s = 2088 \text{ mm}^2/\text{m}. \)

\[ A_{s\min} = 1.4 b d / f_y = 1.4 \times 1000 \times 600 /400 \approx 2100 \text{ mm}^2/\text{m} \quad \text{ACI 318 - 10.5.1} \]

Use \( \phi 25 @ 510/2100 = 0.24 \text{ m} \)

### 4. Check Development Length

\[ l_d = \left( \frac{9 f_y}{\sqrt{f_c'} \sqrt{\frac{\alpha \beta \gamma \lambda}{d_b}}} \right) d_b \quad \text{ACI 318 - 12.2.2} \]

ACI Commentary

In all structures with normal weight concrete, uncoated reinforcement, No. 22 or larger, bottom bars the equations reduce to

\[ l_d = d_b f_y / 1.7 \sqrt{f_c'} \]

as long as minimum cover of \( d_b \) is provided along with a minimum clear spacing of \( 2d_b \) or a minimum clear cover of \( d_b \) and a minimum clear spacing of \( d_b \) are provided along with minimum ties or stirrups. The penalty for spacing bars closer or providing less cover is the requirement that \( l_d = d_b f_y / 1.1 \sqrt{f_c'} \)

Critical section for development is the same as that for maximum moment (at face of column)

\[ l_d = 25 \times 400 / 1.7 \sqrt{25} = 1176 \text{ mm} > 300 \text{ mm} \quad \text{ACI 318 - 12.2.1} \]

Since \( l_d = 1176 \text{ mm} \). is less than the available embedment length (2000 – 750/2 – 75 = 1550) in the long column direction, the bars are fully developed.

### 5. Check Bearing Stresses at Interface of column and footing

\( f_c' \) column = 35 MPa

\( f_c' \) footing = 25 MPa

\( P_u = 4085 \text{ kN} \)

Design bearing force capacity of the concrete of column is

\[ \phi (0.85 f_c' A_1) = 0.65 (0.85 \times 35 \times 0.3 \times 0.75) = 4350 \text{ kN} > P_u \quad \text{OK} \quad \text{ACI 318 - 9.3} , 10.17 \]

Design bearing force capacity of the concrete of footing = \( \sqrt{A_2 / A_1} (0.85 f_c' A_1) \)
A_2 = (0.3 + 2\times 0.5) \times (0.75 + 2\times 0.5) = 6.325 \text{ m}^2 \quad \sqrt{6.325/0.225} = 5.3 \text{ use 2}

Design bearing force capacity of the concrete of footing

\[ = 2 \times 0.65(0.85 \times 25 \times 0.225) = 6215\text{kN} > P_u \text{ OK} \]

6. Required Dowel Bars between Column and Footing

Even though bearing strength on the column and footing concrete is adequate to transfer the factored loads, a minimum area of reinforcement is required across the interface.

\[ A_{s,\text{min}} = 0.005 \times A_1 \quad \text{ACI 318 – 15.8.2.1} \]
\[ A_{s,\text{min}} = 0.005 \times 300 \times 750 = 1125 \text{ mm}^2 \]

Use 4φ 19 mm \quad A_s = 1136 \text{ mm}^2

7. Development of Dowel Reinforcement in Compression:

\text{In Column}

\[ \ell_{dc} = (0.02 f_y / \sqrt{f'_c}) d_b \geq (0.0003 f_y) d_b \quad \text{ACI 318 – 12.3.2} \]

For φ 19 mm = \ell_{dc} = (0.24 \times 400 / \sqrt{35}) 19 \geq (0.043 \times 400) 19

\[ 308 \text{ mm} \quad 326 \text{ mm governs} \]

\text{In Footing}

\[ \ell_{dc} = (0.02 f_y / \sqrt{f'_c}) d_b \geq (0.0003 f_y) d_b \quad \text{ACI 318 – 12.3.2} \]

For φ 19 mm = \ell_{dc} = (0.24 \times 400 / \sqrt{25}) 19 \geq (0.043 \times 400) 19

\[ 365 \text{ mm governs} \quad 326 \text{ mm} \]

Available depth of footing is larger than required development length. OK
Example:
Design the spread footing shown?
Column size = 0.4× 0.4 m with 4 bars φ 30 mm
P_D = 350 kN   \( P_L = 450 \text{ kN} \)
\( f_c' = 30 \text{ MPa} \) \( f_y = 400 \text{ MPa} \)

Solution:

1. Determination of Base Area
\( q_a = q_u + q'/3 = 200 + 0.3 \times 1.2 \times 18 = 206.5 \text{ kN} \) Use 200kN (for factor of safety =3) As a net \( q_a \)
\( A = 800/200 = 4 \text{ m}^2 \) \( B = 2 \text{ m} \)

2. Check immediate settlement
\( E_{\text{stiff clay}} = 1000 \) \( S_u = 1000 \times 200/2 = 100 \text{ MPa} \)
\( E_{\text{sand}} = 500(N_{55} +15) \)
\( E_{\text{sand above water table}} = 500(25 \times 70/55 +15 = 23.4 \text{ MPa} \)
\( E_{\text{sand below water table}} = 500(30 \times 70/55 +15 = 26.6 \text{ MPa} \)
\( E_{\text{av.}} = 100 \times 4.8 + 23.4 \times 3 + 26.6 \times 2.2 /10 = 60.9 \text{ MPa} \)
Use \( \mu = 0.35 \)
For \( H/B' = 10/1 =10 \) \( D/B = 1.2/2 = 0.6 \)
\( l_1 = 0.498 \) \( l_2 = 0.016 \) \( l_r = 0.75 \)
\( S_i = 200x \times (1-0.35^2/60.5) \times (0.498 + (1 -2x0.35/1-0.35)0.016) \times 0.75 /1000 = 4.4 \text{ mm} \)
This is not critical, also consolidation settlement can be ignored because water table is at level below the clay layer.

2. Find Soil Reaction for Strength Design
\( P_u = 1.4 \times 350 +1.7 \times 450 = 1255 \text{ kN} \)
\( q_s = 1255/ 4 = 314 \text{ kPa} \)

Find Footing Thickness Using Shear Criteria
2. Rectangular Footing

Necessity for rectangular footing

Space limitation, so square footing cannot be used

Presence of overturning moment, so rectangular produces more economical design

Example:

Design a rectangular reinforced concrete footing for the given data below

Column size = 0.4 × 0.4 m with 8 bars $\phi$ 25 mm

$P_D = 1100$ kN  $P_L = 1000$ kN

$f_{c'} = 35$ MPa  $f_y = 400$ MPa

Footing  $f_{c'} = 25$ MPa  $f_y = 400$ MPa

Net $q_{all} = 250$ kPa

Solution:

1. Find area of the base

$A = (1100 + 1000)/250 = 8.4$ m$^2$  Assume $B = 2.1$ m gives $L = 4$ m

2. Find thickness and reinforcement

$q_s = (1.4 \times 1100 + 1.7 \times 1000)/8.4 = 386$ kPa

Wide beam shear , long direction

$\phi V_c = 0.85 \times 0.17 \sqrt{f_{c'} b_w d} = 0.85 \times 0.17 \sqrt{25 \times 2.1 \times d \times 1000} = 1517d$

$V_u = 386 \times 2.1 \times (2- d - 0.4/2) = 810 (1.8 - d)$

$V_u = \phi V_c$

$1458 - 810d = 1517d$

$d = 0.63$ m

Check two way shear  $\beta = 1$

$V_c/\sqrt{f_{c'}} b_0 d$

(a) $= 0.17(1+2/1) = 0.51$

(b) $= 0.33$

$use\ 0.33$
\[
\phi V_c = 0.85 \times 0.33 \frac{V_c}{b_0d} = 0.85 \times 0.33 \times 5 \times 4(0.4+d) d \times 1000
\]
\[
= 5610 (0.4d + d^2)
\]
\[
V_u = 386 (8.4 - (0.4+d)^2) = 386 (8.24 - 0.8d - d^2)
\]
\[
\phi V_c = V_u
d = 0.55 \text{ m}
\]

Find Flexural Reinforcement

Short Direction Steel

\[
M_u = q_s \frac{L^2}{2} = 386 \times (1.05- 0.2)^2 /2 \approx 140 \text{ kN m/m}
\]

ACI 318 - 15.4.2 - c

\[
M_u = \phi A_s f_y (d - 0.5 A_s f_y / 0.85 f_c')
\]

\[
, \phi = 0.9
\]

ACI 318 – 9.3.2

\[
140 = 0.9 \times A_s \times 400 (0.63 - 0.5 \times 400 / 0.85 \times 25) \times 1000
\]

\[
A_s = 623 \text{ mm}^2/\text{m}.
\]

\[
A_{s \text{ min}} = 1.4 \frac{b d}{f_y} = 1.4 \times 1000 \times 630 /400 = 2205 \text{ mm}^2/\text{m}
\]

ACI 318 – 10.5.1

\[
A_s \text{ middle band} = 4 \times 2205 (2/(4/2.1 +1) = 8820 \ (2/2.9) = 6082 \text{ mm}^2
\]

\[
= 6082/2.1 \text{ m (band width)} = 2896 \text{ mm}^2/\text{m}
\]

Use \( \phi 25 @ 510/2896 = 0.176 \text{ m} \) say 175 mm

The remaining area of steel = 8820 – 6082 = 2738 mm, this have to be distributed

outside middle band

\[
2738/1.9 \text{ m} = 1441 \text{ mm}^2/\text{m}
\]

Use \( \phi 25 @ 510/2205 = 0.23 \text{ m} \) 230 mm

Long Direction Steel

The steel of the long direction may be laid in a lower layer of the bottom mesh to achieve some saving in the steel mass, so d = 0.63+ (d_b 0.025) = 0.65 m

\[
M_u = q_s \frac{L^2}{2} = 386 \times (2- 0.2)^2 /2 = 625 \text{ kN m/m}
\]

ACI 318 - 15.4.2 - c

\[
M_u = \phi A_s f_y (d - 0.5 A_s f_y / 0.85 f_c')
\]

\[
, \phi = 0.9
\]

ACI 318 – 9.3.2

\[
625 = 0.9 \times A_s \times 400 (0.65 - 0.5 \times 400 / 0.85 \times 25) \times 1000
\]

\[
A_s = 2786 \text{ mm}^2/\text{m} \geq A_{s \text{ min}}
\]

Use \( \phi 25 @ 510/2786 = 0.183 \text{ m} \) use 180 mm

Total thickness of footing (h) = 0.65+0.025/2+0.050m (cover casting on blinding)= 0.725 m
3. Wall Footing

Design of a wall footing consists in providing a depth adequate for wide-beam shear (which will control as long as \( d \leq 2/3 \times \text{footing projection} \)). The remainder of the design consists in providing sufficient reinforcing steel for bending requirements of the footing projection. Longitudinal steel is required to satisfy shrinkage requirements. Longitudinal steel will, in general, be more effective in the top of the footing than in the bottom. The contact pressure is usually on the order of 17 to 25 kPa including the wall weight.

Example:
Design the wall footing for the following data. Wall load consists in 140 kN/m (\( D = 100, \ L = 40 \) kN/m) including wall and roof contribution. 
\( f_{c'} = 21 \text{ MPa} \quad f_y = 400 \text{ MPa} \quad \text{Net} \ q_a = 200 \text{ kPa} \)
Wall of concrete block 200 X 300 X 400 mm

Solution:

Find Footing Width
\( B = 140/200 = 0.7 \text{ m} \)

Find Thickness
\( q_s = (1.4 \times 100 + 1.7 \times 40) / 0.7 = 297 \text{ KPa} \)
Wide beam shear
\( \phi V_c = 0.85 \times 0.17 \sqrt{f_{c'} b_w d} = 0.85 \times 0.17 \sqrt{21 \times 1 \times d \times 1000} = 662 \text{ d} \)
\( V_u = 297 \times 1 \times (0.7/2 – 0.075) = 81.7 \text{ kN} \)
\( V_u = \phi V_c \quad d = 0.123 \text{ m} + 0.07 \text{ cover} = 0.2 \text{m} > 0.15 \text{m} \quad \text{ACI 318-15.7} \)

Find Flexural Reinforcement (Transverse Steel)
\( M_u = q_s L^2/2 = 297 \times (0.275)^2/2 = 11.23 \text{ kN m/m} \)
\( M_u = \phi A_s f_y \ (d - 0.5 A_s f_y/0.85 f_{c'}) \quad , \phi = 0.9 \)
\( 11.23 = 0.9 \times A_s \times 400 \times (0.123 - 0.5 \times A_s \times 400 / 0.85 \times 21) \times 1000 \)
\( A_s = 259 \text{ mm}^2/m. \)
\( A_{s \min} = 1.4 \times d / f_y = 1.4 \times 1000 \times 123 / 400 = 431 \text{ mm}^2/m \quad \text{Control} \)
Use \( \phi 13 @ 129/431 = 0.3 \text{ m} < \text{max. spacing} 3h \quad \text{ACI 318-10.5.4} \)

Temperature and Shrinkage Reinforcement (Longitudinal Steel)
\( A_s = 0.0018 \times 200 \times 700 = 252 \text{mm}^2 \)
Use 4 bars \( \phi 10 \) provide \( 4 \times 71 \text{mm}^2 = 284 \text{mm}^2 \)
Example:
A36 cm masonry wall carries a load \( P_D = 150\text{kN/m}, P_L = 120\text{kN/m} \), if the allowable bearing capacity is 200 \( \text{kN/m}^2 \), \( f_{c'} = 21 \text{ MPa}, f_y = 400\text{MPa} \).
1. Design a plain concrete footing.
2. Design a reinforced concrete footing.

Solution:

1. Find area of footing
\[ B = \frac{((150+120) +5\% \times 270)}{200} = 1.4 \text{ m} \]
\[ q_s = \frac{(1.4 \times 150 + 1.7 \times 120)}{1.4} = 296 \text{ kPa} \]

1. Plain Concrete Footing
Find Thickness
\[ f_t = 0.42\phi \sqrt{f_{c'}} = 0.42 \times 0.65 \times \sqrt{21} = 1.25 \text{ MPa} \]
\[ M_u = \frac{w l^2}{2} = 296 \times (0.7 - 0.36/4)^2 /2 = 55 \text{ kN.m/m} \]
\[ f_t = \frac{M}{S}, S = bd^2/6 \]
\[ d = \sqrt{\frac{55 \times 6}{(1.25 \times 1 \times 1000)}} = 0.52 \text{ m} \]
Overall thickness \( h = d + 50 \text{ mm} = 0.52 + 0.05 = 0.57 \text{ m} \) Use 0.60m

1. Reinforced Concrete Footing
Wide beam shear
\[ \phi V_c = 0.85 \times 0.17 \sqrt{f_{c'}} b_w d = 0.85 \times 0.17 \sqrt{21} \times 1 \times d \times 1000 = 662 \text{ d} \]
\[ V_u = 296 \times 1 \times (1.4/2 - 0.36/4) = 180.5 \text{ kN} \]
\[ V_u = \phi V_c \]
\[ d = 0.27 \text{m} + 0.07 \text{ cover} = 0.35 \text{m} > 0.15 \text{m} \]

Find Flexural Reinforcement (Transverse Steel)
\[ M_u = 55 \text{ kN.m}/ \]
\[ M_u = \phi A_s f_y \left( d - 0.5 A_s f_y / 0.85 f_{c'} \right) \]
\[ 55 = 0.9 \times A_s \times 400 \left(0.27 - 0.5 A_s \times 400 / 0.85 \times 21\right) 1000 \]
\[ A_s = 580 \text{ mm}^2/\text{m}. \]
\[ A_{s_{\text{min}}} = 1.4 b d / f_y = 1.4 \times 1000 \times 270 / 400 = 945 \text{ mm}^2/\text{m} \] Control
Use \( \phi 16 @ 200/945 = 0.2 \text{ m} < \text{max. spacing 3h} \) ACI 318- 10.5.4

Temperature and Shrinkage Reinforcement (Longitudinal Steel)
\[ A_s = 0.0018 \times 350 \times 1400 = 882 \text{ mm}^2 \]
Use 7 bars \( \phi 13 \) provide 7x129 mm\(^2\) = 903 mm\(^2\)
Combined Footing

Combined footing: a footing supports a line of two or more columns, it may have either rectangular or trapezoidal shape or be a series of pads connected by narrow rigid beams called a strap footing.

At a certain circumstances, it may not be possible to place columns at the center of a spread footing if they are near the property line, near mechanical equipment locations, irregularly spaced, or the space between excavations is less than the smaller width of adjacent footings. Columns located off-center will usually result in a nonuniform soil pressure, in which the combined footing is the solution.

Assumptions:
1. Uniform planer distribution of soil pressure.
2. Uniform soil condition.

Design procedure:
1. Locate the point of application of the resultant of all applied loads.
2. Compute the area of footing using net allowable bearing capacity and working loads.
   a. Uniform soil pressure if resultant does coincide with the center of footing area; \( q = \frac{P}{A} \).
   b. Linear distribution if resultant doesn’t coincide with the center of footing area;
      \[ q = \frac{P}{A} + \frac{P_e c}{I} \]
3. Using factored load to find the ultimate soil pressure.
4. Construct the shear force and bending moment diagrams
5. Find depth of footing from the ultimate strength at critical sections.
6. Find reinforcement and details according to code requirement.

1. Rectangular Footing

Example:
Design a combined footing for the given Condition
Column A (0.4x 0.4) \( P_D = 650 \text{ kN} \) \( P_L = 400 \text{ kN} \)
Column B (0.4x 0.4) \( P_D = 850 \text{ kN} \) \( P_L = 500 \text{ kN} \)
\( f_{c'} = 21 \text{ MPa} \) \( f_y = 400 \text{ MPa} \) \( q_a = 150 \text{ kPa} \)

Solution:
1. Find area of footing
   Find location of load resultant \( \Sigma M_A = 0 \)
   \[ X = 4.8 \frac{(850+500)}{(850+500+650+400)} = 2.7 \text{m} \]
   Length of footing \( L = 2 \times (2.7 + 0.4/2) = 5.8 \text{m} \)
   Width of footing \( B = (1.05 \times 2400)/(150 \times 5.8) = 2.9 \text{m} \)

2. Find thickness and reinforcement
   \( P_u = 1.4(650+850)+1.7(400+500) = 3630 \text{kN} \)
Pu A = 1590kN           Pu B = 2040kN

\[ q_s = \frac{3630}{(5.8 \times 2.9)} = 216kN/m^2 \]

Two way shear at column A

\[ \phi V_c = 0.85 \times 0.33 \sqrt{f_{c'}} b_o d = 0.85 \times 0.33 \times \sqrt{21} (3 \times 0.4 + 2d) d \times 1000 = 2570 \ (0.6d + d^2) \]

\[ V_u = 1590 - 216(0.4+d)(0.4+d/2) = 1512 - 86.4d - 216 d^2 \]

\[ V_u = \phi V_c \]

\[ d = 0.50m \]

Two way shear at column B

\[ \phi V_c = 0.85 \times 0.33 \sqrt{f_{c'}} b_o d = 0.85 \times 0.33 \times \sqrt{21} \times 4(0.4+d) d \times 1000 = 5141 \ (0.4d + d^2) \]

\[ V_u = 2040 - 216(0.4+d)(0.4+d) = 2005 - 173d - 216 d^2 \]

\[ V_u = \phi V_c \]

\[ d = 0.44m \]

Wide beam shear

Column A

\[ V_u = 1590 - 626(0.4+d) = 1340 - 626d \ kN \]

Column B

\[ V_u = 2040 - 626(0.4+0.6+d) = 1414 - 626d \ kN \]

Governs

\[ \phi V_c = 0.85 \times 0.17 \sqrt{f_{c'}} b w d = 0.85 \times 0.17 \sqrt{21} \times 2.9 \times d \times 1000 = 1620 \ d \]

\[ V_u = \phi V_c \]

\[ d = 0.63 \ m \quad +0.05 \text{ cover} \quad \text{Use } h= 0.70m > 0.15m \text{ minimum} \]

Long Direction Steel

Maximum moment at point where shear equal to zero

\[ 1590= 626(0.2+x) \quad x= 2.34 \text{ from center of column A} \]

\[ M_u = 1590 \times 2.34 - 626(2.54)^2/2 = 1701 \ kN.m / 2.9 = 586 \ kN.m/m \]

\[ M_u = \phi A_s f_y \ (d - 0.5 A_s f_y/0.85 f_{c'}) \]

\[ 586 = 0.9 \times A_s \times 400 \times (0.65 - 0.5 A_s \times 400) / 0.85 \times 21 \times 1000 \]

\[ A_s = 2625 \text{ mm}^2/m. \quad \text{Control} \]

\[ A_{s\text{min}} = 1.4 \times b \times f_y = 1.4 \times 1000 \times 650 / 400 = 2275 \text{ mm}^2/m \]

\[ \text{Use } \phi 25 @ 510/2625 = 0.2m \quad 200 \text{ mm Top steel} \]

Short Direction Steel

Effective footing width under column A = column width+0.75d

\[ = 0.4 + 0.75 \times 0.65 = 0.887m \approx 0.90m \]

\[ q_s = 1590/2.9 = 548kN/m \]

\[ M_u = 548 (1.25)^2/2 = 428 \ kN.m \quad \text{use } d=0.63 \]

\[ M_u = \phi A_s f_y \ (d - 0.5 A_s f_y/0.85 f_{c'b}) \]

\[ 428 = 0.9 \times A_s \times 400 \times (0.63 - 0.5 A_s \times 400) / 0.85 \times 21 \times 0.9) \times 1000 \]

\[ A_s = 1963 \text{ mm}^2 \]
Effective footing width under column B = column width + 2 x 0.75d
= 0.4 + 2 x 0.75 x 0.65 = 1.375 m

Calculate reinforcement as before

2. Trapezoid Footing

In case where the applied load carried by the exterior column is larger than that carried by the interior one, the resultant will be closer to the exterior column and doubling this distance doesn’t reach the exterior column and no rectangular shape combined footing can be constructed. A trapezoid shape combined footing of area having center coincide with the resultant of applied forces will result in a linear soil pressure distribution.

Trapezoid geometry equations

\[ A = \frac{L(a+b)}{2} \]

\[ x' = \frac{L}{3} \left( \frac{2a+b}{a+b} \right) \]

\[ \frac{L}{3} < x' < \frac{L}{2} \]

Example:

Design a combined footing for the given condition.

Column a (0.4 x 0.4) \( P_D = 800 \text{ kN} \) \( P_L = 600 \text{ kN} \)

Column b (0.6 x 0.6) \( P_D = 1100 \text{ kN} \) \( P_L = 900 \text{ kN} \)

\( f_c' = 21 \text{ MPa} \) \( f_y = 400 \text{ MPa} \) \( q_a = 200 \text{ kPa} \)

center to center of column = 6 m

Solution:

1. Find area of footing

\[ A = \frac{(800+600+1100+900) \times 1.05}{200} = 17.85 \text{ m}^2 \]

Find location of load resultant \( \Delta M_b = 0 \)

\[ X = \frac{6(800+600)}{(800+600+1100+900)} = 2.47 \text{ m} \]

\[ X' = 2.47 + 0.3 = 2.77 \]

\[ 2.77 = \frac{6.5}{3} \left( \frac{2a+b}{a+b} \right) \]

\[ 17.85 = \frac{6.5(a+b)}{2} \]

Solving 1 & 2 simultaneously gives \( a = 1.54 \text{ m} \) Use 1.5 m, \( b = 3.96 \text{ m} \) Use 4 m

2. Find thickness and reinforcement

\( P_u a = 2140 \text{ kN} \) \( P_u b = 3070 \text{ kN} \)

\( P_u \text{ total} = 2140+3070 = 5210 \text{ kN} \)

\( q_s = \frac{5210}{(17.85)} = 292 \text{ kN/m}^2 \)

\( q_s = 292 \times 4 = 1168 \text{ kN/m along footing length at side b} \)
qs = 292 x 1.5 = 438 kN/m along footing length at side a

**Two way shear at column b**

\[ \phi V_c = 0.85 \times 0.33 \sqrt{f_{c'}} \times b_w \times d = 0.85 \times 0.33 \times \sqrt{21 (3 \times 0.6 + 2d)} \times d \times 1000 = 2570 (0.9d + d^2) \]

\[ V_u = 3070 - 292 (0.6 + d) (0.6 + d/2) = 2965 - 263d - 146d^2 \]

\[ V_u = \phi V_c \quad d = 0.67 \text{ m} \]

**Two way shear at column a**

\[ \phi V_c = 0.85 \times 0.33 \sqrt{f_{c'}} \times b_w \times d = 0.85 \times 0.33 \times \sqrt{21 (3 \times 0.4 + 2d)} \times d \times 1000 = 2570 (0.6d + d^2) \]

\[ V_u = 2140 - 292 (0.4 + d) (0.4 + d/2) = 2093 - 175d - 146d^2 \]

\[ V_u = \phi V_c \quad d = 0.62 \text{ m} \]

**Wide beam shear**

**Column b**

\[ V_u = 3070 - 292 (0.6 + d) (3.88 - 0.19d) = 2390 - 1098d + 55.5d^2 \text{kN} \]

\[ \phi V_c = 0.85 \times 0.17 \sqrt{f_{c'} b_w d} = 0.85 \times 0.17 \sqrt{21 (3.77 - 0.38d)} \times d \times 1000 = 2496d - 252d^2 \]

\[ V_u = \phi V_c \quad d = 0.707 \text{ m} \]

**Column a**

\[ V_u = 2140 - 292 (0.4 + d) (1.58 - 0.19d) = 1955.5 - 439d + 55.5d^2 \text{kN} \]

\[ \phi V_c = 0.85 \times 0.17 \sqrt{f_{c'} b_w d} = 0.85 \times 0.17 \sqrt{21 (1.65 + 0.38d)} \times d \times 1000 = 1092.6d + 251.6d^2 \]

\[ V_u = \phi V_c \quad d = 1.12 \text{ m} \text{ Control Use } h = 1.2 \text{ m} \]

Construct first degree load distribution, second degree shear force and third degree bending moment diagrams to determine reinforcement different locations along long side due to different section width and short directions and the other details, similarly as done in previous example.

### 3. Strap (or Cantilever) Footing

A strap footing is used to connect an eccentrically loaded column footing to an interior column as shown in figure. The strap is used to transmit the moment caused from eccentricity to the interior column footing so that a uniform soil pressure is computed beneath both footings.

The strap footing may be used in lieu of a combined rectangular or trapezoid footing if the distance between columns is large and/or the allowable soil pressure is relatively large so that the additional footing area is not needed. Three basic considerations for strap footing design are these:

1. Strap must be rigid—perhaps I strap/ I footing > 2. This rigidity is necessary to control rotation of the exterior footing.
2. Footings should be proportioned for approximately equal soil pressures and avoidance of large differences in B to reduce differential settlement.
3. Strap should be out of contact with soil so that there are no soil reactions to modify the design assumptions. It is common to neglect strap weight in the design. Check depth to span (between footing edges) to see if it is a deep beam (ACI Art. 10-7).

Example:
Design a strap footing for the given data?

\[ q_{a \text{ net}} = 120 \text{ kPa} \]

1. Find Area of Footing
Find location of load resultant \( \Sigma M_a = 0 \)
\[ X = 6.2 \frac{(320+260)}{(320+260+500+400)} = 2.43 \text{ m} \]
\[ X' = 2.43 + 0.20 = 2.63 \text{ m}, \] this is close to \( L/3 \) which gives triangle shape combined footing.

Assume \( e \)
Approximate trial
\[ A = \frac{580}{120} = 4.83 \text{ m}^2 \quad B = 2.2 \text{ m} \quad B = 2(w/2 + e) \quad e = B/2 - w/2 = 1.1 - 0.2 = 0.9 \text{ m} \] Use 1m

\[ \Sigma M_a = 0 \quad R_1 \times 5.2 - 980 \times 6.2 = 0 \quad R_1 = 691.5 \text{ kN} \]
\[ \Sigma M_b = 0 \quad 900 \times 6.2 - 691.5 \times 1 - R_2 \times 6.2 = 0 \quad R_2 = 788.5 \text{ kN} \]

Check by \( \Sigma F_y = 0 \)
\[ 691.5 + 788.5 - 580 - 900 = 0 \] OK

\[ A_1 = \frac{R_1}{120} = \frac{691.5}{120} = 5.76 \text{ m}^2 \quad L = \frac{5.76}{2.4} = 2.4 \text{ m} \]

\[ A_2 = \frac{788.5}{120} = 6.57 \text{ m}^2 \quad \text{assume } B = 2.4 \quad L = 2.75 \]

2. Find Thicknesses and Reinforcement

\[ P_{ua} = 1.4 \times 500 + 1.7 \times 400 = 1380 \text{ kN} \]
\[ P_{ub} = 1.4 \times 320 + 1.7 \times 260 = 980 \text{ kN} \]
\[ \Sigma M_a = 0 \quad R_1 \times 5.2 - 980 \times 6.2 = 0 \quad R_1 = 1168.5 \text{ kN} \]
\[ q_s = \frac{1168.5}{(2.4 \times 2.4)} = 203 \text{ kN/m}^2 = 487 \text{ kN/m} \]
\[ \Sigma M_b = 0 \quad 1380 \times 6.2 - 1168.5 \times 1 - R_2 \times 6.2 = 0 \quad R_2 = 1191.5 \text{ kN} \]
\[ q_s = \frac{1191.5}{(2.4 \times 2.75)} = 181 \text{ kN/m}^2 = 435 \text{ kN/m} \]
Design strap for shear force =188kN and negative moment 746 kN.m assuming b=0.4m and d=2.5 depth of footing

Raft “Mat” Foundation

Introduction
A raft foundation is a large concrete slab used to interface one or more column in several lines, with the base soil, which also may be supported by piles.
A raft foundation may be used where;
   1. The soil has low bearing capacity.
   2. The soil containing soft and hard pockets.
   3. Basement floor is required
   4. The load from the periphery of building has to be distributed over the entire area.

Raft foundations are commonly used to overcome both total and differential settlement problems via controlling measurements
1. Large foundation produces low soil contact pressure.
2. Floatation effect; replacing soil weight partially or totally by raft and superstructure loading.
3. Bridging effects attributable to
   a. Mat rigidity.
   b. Contribution of superstructure rigidity to the mat.
4. Allowing somewhat larger settlements, say, 50 instead of 25 mm

Types of Raft foundation
1. Flat Plate; when columns spacing are small and uniform or columns with small or moderate loads.
2. Plate thickened under columns; to provide sufficient shear strength under large column’s load.
3. Plate with pedestals; where top surface is not required as a floor.
4. Waffle slab; when spacing of columns are large with unequal load.
Bearing Capacity of Raft Foundation

For Cohesive Soil $\phi=0$, $N_q=1$

$$q_{ult} = C \ N_c \ S_c \ S_d + q' \quad q_{ult\ net} = C \ N_c \ S_c \ S_d + q' - q' = C \ N_c \ S_c \ S_d$$

For Raft with Basement

$$q_{ult\ net} = C \ N_c \ S_c \ S_d + q' \quad \text{where} \quad q' = \gamma D_f$$

For Settlement of Raft $\Delta \sigma_v = q_c \gamma D_f$

For Granular Soil

$$q_a = \frac{N}{F_2} K_d \quad \text{For mat foundation}$$

Where $K_d = 1 + 0.33 D/B \leq 1.33$

$q_a$ = allowable bearing pressure for $\Delta H_o = 25$-mm or 1-in. settlement, kPa, for any other value of settlement $\Delta H_i$, $q_a = \Delta H_i/\Delta H_o \times q_a$

$N_55$ is the statistical average value of blows for the footing influence zone of about 0.5B above footing base to at least 2B below. To convert $N'_{70} \times 70 = N_{55} \times 55$

Example:

Determine the net ultimate bearing capacity of a mat foundation measuring 14 m X 9m on a saturated clay with $C_u = 95$ kPa, $\phi=0$, and $D_f = 2$ m?

Solution

$$q_{ult\ net} = 5.14 \times 95 \ (1 + 0.195 \times 9/14) \ (1+0.4 \times 2/9) = 598 \text{ kPa}$$

Example:

What will be the net allowable bearing capacity of a mat foundation with dimensions of 15 m x 10 m constructed over a sand deposit? Here, $D_f = 2$ m, the allowable settlement is 25 mm, and the average penetration number $N_{55} = 10$

Solution

$$q_{ult\ net} = 10/0.08 \ (1+0.33 \times 2/10) = 133 \text{ kPa}$$

Design Assumptions

1. Raft is infinitely rigid.
2. Soil pressure distribution is in a straight line, so that the centroid of the soil pressure coincides with the line of action of the resultant force acting on the footing.

$q_s = \Sigma P/A = \text{total column loads/Area}$

In case of eccentricity

$$q_s = \Sigma P/A \pm M_y \times l_y \pm M_x y/l_x$$
Example:

Using the conventional method, design the raft foundation below for the condition of loading shown and $f_{c'}=25$MPa, $f_y=400$MPa, $q_{a\text{ net}}=5$kPa, neglect footing weight

\[ \Sigma P= 2(38+50) +44+40+2(150+150+112) =1084 \text{ kN} \]
\[ ex =((44+112+112+40)6 - (38+150+150+38)6 )/1084= - 0.38 \text{ m} \]
\[ My=1084x0.38=412 \text{ kN.m} \]
\[ ey =(44x10.8-40x10.8)/1084=0.04m \]
\[ Mx=1084x0.04=43 \text{ kN.m} \]
\[ Ix=12.6x22.2^3/12=11488 \text{ m}^4 \]
\[ Iy= 22.2x12.6^3/12=3700 \text{ m}^4 \]

\[ \Sigma P_u = 2(58+76) +60.8+67.6+4x228+2x187.1) =1682.6 \text{ kN} \]
\[ ex =((67.6+2x187.1+60.8)6 - (58+2x228+58)6 )/1682.6= - 0.25 \text{ m} \]
\[ ey =(67.6x10.8-60.8x10.8)/1682.6=0.04m \]
Check the soil pressure at corner points in order not to exceed the allowable bearing capacity

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Using factored load to complete the structural design of the raft.

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The raft can be analyzed and design using different procedures

Find thickness,
Column H
\[ \phi V_c = 0.85 \times 0.33 \times \frac{V_{f'}}{b_d} = 0.85 \times 0.33 \times \sqrt{25} \times 4(0.4 + d) \times 1000 = 1402.5 \times (0.4d + d^2) \]
\[ V_u = 228 - (0.4 + d)^2 = 227 - 4.8d - 6d^2 \]
\[ \phi V_c = V_u \quad d = 0.25m \]

Column C
\[ \phi V_c = 0.85 \times 0.33 \times \frac{V_{f'}}{b_d} = 0.85 \times 0.33 \times \sqrt{25} \times 2(0.5 + d/2) \times 1000 = 1402.5 \times (d + d^2) \]
\[ V_u = 67.6 - 6.73(0.5 + d/2)^2 = 66 - 3.36d - 1.68d^2 \]
\[ \phi V_c = V_u \quad d = 0.05m \]