Chapter Two

Reservoir Properties

Porosity

The porosity of a rock is a measure of the storage capacity (pore volume) that is capable of holding fluids. Quantitatively, the porosity is the ratio of the pore volume to the total volume (bulk volume). This important property is determined mathematically by the following generalized relationship:

$$\phi = \frac{\text{pore volume}}{\text{bulk volume}}$$

Where $V_p$: pore volume; $V_{ma}$: matrix volume and $V_b$: bulk volume.

Also, the porosity is an intensive porosity describing the fluid storage capacity of rock.

$$\rho_b = \rho_f \phi + \rho_{ma}(1 - \phi)$$

$$\rho_{bdry} = \rho_{ma}(1 - \phi)$$

$$\rho_b = \sum_{i=1}^{i=N} c_i \rho_i$$

As the sediments were deposited and the rocks were being formed during past geological times, some void spaces that developed became isolated from the other void spaces by excessive cementation. Thus, many of the void spaces are interconnected while some of the pore spaces are completely isolated. This leads to two distinct types of porosity, namely:

- **Absolute porosity**

  The absolute porosity $\phi_a$ (or total porosity $\phi_t$) is defined as the ratio of the total pore space in the rock to that of the bulk volume. A rock may have considerable absolute porosity and yet have no conductivity to fluid for lack of pore interconnection. The absolute porosity is generally expressed mathematically by the following relationships:
\[ \phi_a = \frac{\text{total pore volume}}{\text{bulk volume}} \]

or

\[ \phi_a = \frac{\text{bulk volume} - \text{grain volume}}{\text{bulk volume}} \]

where \( \phi_a \) = absolute porosity.

**Effective porosity**

The effective porosity \( (\phi_e) \) is the percentage of interconnected pore space with respect to the bulk volume, or

\[ \phi_e = \frac{\text{interconnected pore volume}}{\text{bulk volume}} \]

where \( \phi_e \) = effective porosity

The effective porosity is the value that is used in all reservoir engineering calculations because it represents the interconnected pore space that contains the recoverable hydrocarbon fluids (contains mobile fluid).

Where:

- Very clean sandstones: \( \phi_e \rightarrow \phi_t \)
- Poorly to moderately well-cemented intergranular materials: \( \phi_t \approx \phi_e \)
- Highly cemented materials and most carbonates: \( \phi_e < \phi_t \)

**Average porosity**

The reservoir rock may generally show large variations in porosity vertically but does not show very great variations in porosity parallel to the bedding planes. In this case, the arithmetic average porosity or the thickness-weighted average porosity is used to describe the average reservoir porosity. A change in sedimentation or depositional conditions, however, can cause the porosity in one portion of the reservoir to be greatly different from that in another area. In such cases, the areal-weighted average or the volume-weighted average porosity is used to characterize the average rock porosity. These averaging techniques are expressed mathematically in the following forms:
Arithmetic average \( \phi = \frac{\Sigma \phi_i}{n} \)
Thickness-weighted average \( \phi = \frac{\Sigma \phi_i h_i}{\Sigma h_i} \)
Areal-weighted average \( \phi = \frac{\Sigma \phi_i A_i}{\Sigma A_i} \)
Volumetric-weighted average \( \phi = \frac{\Sigma \phi_i A_i h_i}{\Sigma A_i h_i} \)

where \( n = \) total number of core samples
\( h_i = \) thickness of core sample \( i \) or reservoir area \( i \)
\( \phi_i = \) porosity of core sample \( i \) or reservoir area \( i \)
\( A_i = \) reservoir area \( i \)

One important application of the effective porosity is its use in determining the original hydrocarbon volume in place. Consider a reservoir with an areal extent of \( A \) acres and an average thickness of \( h \) feet. The total bulk volume of the reservoir can be determined from the following expressions:

\[
\text{Bulk volume} = 43,560 \text{ Ah, ft}^3
\]

or

\[
\text{Bulk volume} = 7,758 \text{ Ah, bbl}
\]

where \( A = \) areal extent, acres
\( h = \) average thickness

The reservoir pore volume \( PV \) can then be determined by combining Equations 4-4 and 4-5 with 4-3. Expressing the reservoir pore volume in cubic feet gives:

\[
PV = 43,560 \text{ Ah} \phi, \text{ ft}^3
\]

Expressing the reservoir pore volume in barrels gives:

\[
PV = 7,758 \text{ Ah} \phi, \text{ bbl}
\]

**Note:** The acre is a unit of land area used in the imperial and US customary systems. It is defined as the area of 1 chain (22 yards) by 1 furlong (220 yards), which is exactly equal to \( \frac{1}{640} \) of a square mile, 43,560 square feet, approximately 4,046.856 m\(^2\), or about 40% of a hectare.
Porosity of packing

For a uniform rock grain size, porosity is independent of the size of the grains. A maximum theoretical porosity of 48% is achieved with cubic packing of spherical grains, as shown in Fig. 2a. Rhombohedral packing, which is more representative of reservoir conditions, is shown in Fig. 2b; the porosity for this packing is 26%. If a second, smaller size of spherical grains is introduced into cubic packing (Fig. 2c), the porosity decreases from 48% to 14%. Thus, porosity is dependent on the grain size distribution and the arrangement of the grains, as well as the amount of cementing materials. Not all grains are spherical, and grain shape also influences porosity. A typical reservoir sand is illustrated in Fig. 2d.

Fig. 2 Cubic packing (a), rhombohedral (b), cubic packing with two grain sizes (c), and typical sand with irregular grain shape (d).

Classification of Rocks
Sedimentary rocks are classified to:
- Clastics
- Carbonates
- Evaporates

**Clastic Rocks**
- Consist primarily of silicate minerals
- Are classified on the basis of:
  - Grain size
  - Mineral composition

**Carbonate Rocks**
- Consist primarily of carbonate minerals (i.e. minerals with a $\text{CO}_3^{2-}$ anion group)
- Limestone - Predominately calcite (calcium carbonate, $\text{CaCO}_3$)
- Dolomite - Predominately dolostone (calcium magnesium carbonate, $\text{CaMg(CO}_3)_2$)

![Pie chart showing the percentage distribution of different types of sedimentary rocks. Siltstone and shale (clastic) is ~75%, and Limestone and dolomite is ~14%. Sandstone and conglomerate (clastic) is ~11%.]
Grain-Size Classification for Clastic Sediments

<table>
<thead>
<tr>
<th>Name</th>
<th>Millimeters</th>
<th>Micrometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulder</td>
<td>4,096</td>
<td></td>
</tr>
<tr>
<td>Cobble</td>
<td>256</td>
<td></td>
</tr>
<tr>
<td>Pebble</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Granule</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Very Coarse Sand</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Coarse Sand</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Medium Sand</td>
<td>0.5</td>
<td>500</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>0.25</td>
<td>250</td>
</tr>
<tr>
<td>Very Fine Sand</td>
<td>0.125</td>
<td>125</td>
</tr>
<tr>
<td>Coarse Silt</td>
<td>0.062</td>
<td>62</td>
</tr>
<tr>
<td>Medium Silt</td>
<td>0.031</td>
<td>31</td>
</tr>
<tr>
<td>Fine Silt</td>
<td>0.016</td>
<td>16</td>
</tr>
<tr>
<td>Very Fine Silt</td>
<td>0.008</td>
<td>8</td>
</tr>
<tr>
<td>Clay</td>
<td>0.004</td>
<td>4</td>
</tr>
</tbody>
</table>

(modified from Blatt, 1982)

Sandstone reservoir

- **Framework**
  Sand (and Silt) Size Detrital Grains

- **Matrix**
  Silt and Clay Size Detrital Material

- **Cement**
  Material Precipitated Post-Depositionally, During Burial. Cements Fill Pores and Replace Framework Grains

- **Pores**
  Voids Among the Above Components
**SANDSTONES POROSITY TYPES**

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intergranular (Primary)</td>
<td>Interstitial Void Space Between Framework Grains</td>
</tr>
<tr>
<td>Micropores</td>
<td>Small Pores Mainly Between Detrital Framework Grains or Cement</td>
</tr>
<tr>
<td>Dissolution</td>
<td>Partial or Complete Dissolution of or Authigenic Grains (Can Also Occur Within Grains)</td>
</tr>
<tr>
<td>Fractures</td>
<td>Breakage Due to Earth Stresses</td>
</tr>
</tbody>
</table>

**Carbonate reservoir**

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interparticle</td>
<td>Pores Between Particles or Grains</td>
</tr>
<tr>
<td>Intraparticle</td>
<td>Pores Within Individual Particles or Grains</td>
</tr>
<tr>
<td>Intercrystal</td>
<td>Pores Between Crystals</td>
</tr>
<tr>
<td>Moldic</td>
<td>Pores Formed by Dissolution of an Individual Grain or Crystal in the Rock</td>
</tr>
<tr>
<td>Fenestral</td>
<td>Primary Pores Larger Than Grain-Supported Interstices</td>
</tr>
<tr>
<td>Fracture</td>
<td>Formed by a Planar Break in the Rock</td>
</tr>
<tr>
<td>Vug</td>
<td>Large Pores Formed by Indiscriminate Dissolution of Cements and Grains</td>
</tr>
</tbody>
</table>
Geological Classification of Porosity

Porosity may be classified according to the mode of origin as originally induced. The original porosity is that developed in the deposition of the material, while induced porosity is that developed by some geological process subsequent to deposition of the rock. The intergranular porosity of sandstones and the intercrystalline and oolitic porosity of some limestones typify original porosity. Induced porosity is typified by fracture development as found in shale's and limestones and by the slugs or solution cavities commonly found in limestones. Rocks having original porosity are more uniform in their characteristics than those rocks in which a large part of the porosity is included. For direct quantitative measurement of porosity, reliance must be placed on formation samples obtained by coring. Since effective porosity is the porosity value of interest to the petroleum engineer, particular attention should be paid to the methods used to determine porosity.

**PRIMARY (ORIGINAL) POROSITY**

- Developed at deposition
- Typified by
  - Intergranular pores of clastics or carbonates
  - Intercrystalline and fenestral pores of carbonates
- Usually more uniform than induced porosity
SECONDARY (INDUCED) POROSITY

- Developed by geologic processes after deposition (diagenetic processes)
- Examples
  - Grain dissolution in sandstones or carbonates
  - Vugs and solution cavities in carbonates
  - Fracture development in some sandstones, shales, and carbonates

FACTORS THAT AFFECT POROSITY

PRIMARY
- Particle sphericity and angularity
- Packing
- Sorting (variable grain sizes)

SECONDARY (diagenetic)
- Cementing materials
- Overburden stress (compaction)
- Vugs, dissolution, and fractures
- Clay content

Note: 1-Fracture porosity= ratio of void space within fracture to bulk volume.
2-Dual porosity refers to a system where both matrix and fracture porosity exit.
Porosity is classed in the reservoir as:

- Porosity < 5 poor low
- Porosity = 10 – 20 average
- Porosity = 20 – 30 good
- Porosity > 30 excellent
Example 1

An oil reservoir exists at its bubble-point pressure of 3,000 psia and temperature of 160°F. The oil has an API gravity of 42° and gas-oil ratio of 600 scf/STB. The specific gravity of the solution gas is 0.65. The following additional data are also available:

• Reservoir area = 640 acres
• Average thickness = 10 ft
• Connate water saturation = 0.25
• Effective porosity = 15%

Calculate the initial oil in place in STB.
Step 1. Determine the specific gravity of the stock-tank oil from Equation 2-68.

\[ \gamma_o = \frac{141.5}{42 + 131.5} = 0.8156 \]

Step 2. Calculate the initial oil formation volume factor by applying Standing’s equation, i.e., Equation 2-85, to give:

\[ B_0 = 0.9759 + 0.00012 \left[ 600 \left( \frac{0.65}{0.8156} \right)^{0.5} + 1.25(160) \right]^{1.2} \]

\[ = 1,396 \text{ bbl/STB} \]

Step 3: Calculate the pore volume

Pore volume = 7758 (640) (10) (0.15) = 7447680 bbl

Step 4: Calculate the initial oil in place

Initial oil in place = 7447680 (1-0.25)/1.306 = 4001260 STB

**Note:**

Standing (1981) showed that the oil formation volume factor can be expressed more conveniently in a mathematical form by the following equation:

\[ B_0 = 0.9759 + 0.000120 \left[ R_s \left( \frac{\gamma_g}{\gamma_o} \right)^{0.5} + 1.25(T - 460) \right]^{1.2} \]

where

- \( T \) = temperature, °R
- \( \gamma_o \) = specific gravity of the stock-tank oil
- \( \gamma_g \) = specific gravity of the solution gas
Example 2

Calculate the arithmetic average and thickness-weighted average from the following measurements:

<table>
<thead>
<tr>
<th>Sample</th>
<th>Thickness, ft</th>
<th>Porosity, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>2.1</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>1.1</td>
<td>10</td>
</tr>
</tbody>
</table>

Solution

• Arithmetic average

\[
\phi = \frac{10 + 12 + 11 + 13 + 14 + 10}{6} = 11.67\%
\]

• Thickness-weighted average

\[
\phi = \frac{(1)(10) + (1.5)(12) + (1)(11) + (2)(13) + (2.1)(14) + (1.1)(10)}{1 + 1.5 + 1 + 2 + 2.1 + 1.1} = 12.11\%
\]

Example 3

Suppose a rectangular reservoir is 10 miles long, 3 miles wide and 100 feet thick; and suppose the porosity of a rock sample obtained from the reservoir is 22%. The bulk volume of the reservoir is then

\[
V_b = 10 \times 5280 \times 3 \times 5280 \times 100
\]

\[
= 8.364 \times 10^{10} \text{ ft}^3
\]

and its pore volume is

\[
V_p = 8.364 \times 10^{10} \times 0.22
\]

\[
= 1.840 \times 10^8 \text{ ft}^3
\]

\[
= 3.277 \times 10^7 \text{ bbl}
\]

Note: Units most commonly used in the petroleum industry and their conversion factors are listed in Appendix A.
Example 4

The contour map below depicts porosity variation within a reservoir. The lines connect points of equal porosity. If the reservoir is 10 miles long, 3 miles wide, and 100 feet thick, its average porosity is estimated as follows:

There are 4 sections within the reservoir, the area and average porosity of each are computed to be:

<table>
<thead>
<tr>
<th>Section</th>
<th>Area (ft²)</th>
<th>Avg. Porosity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>155,399,147</td>
<td>12.5</td>
</tr>
<tr>
<td>2</td>
<td>277,261,208</td>
<td>17.5</td>
</tr>
<tr>
<td>3</td>
<td>122,322,723</td>
<td>22.5</td>
</tr>
<tr>
<td>4</td>
<td>14,303,684</td>
<td>25.0</td>
</tr>
<tr>
<td>Total</td>
<td>569,286,762</td>
<td></td>
</tr>
</tbody>
</table>

Since the reservoir thickness is uniform, section areas only are used in the computation.

\[
\bar{\phi} = \left( \frac{\sum \phi_i A_i}{A} \right) / A
\]

\[
= 9,904,413,847 / 569,286,762
\]

\[
= 17.4
\]
Example 5
An irregular piece of sandstone is 35.25 grams in mass. When coated with varnish, its mass increased to 36.55 grams. Compute the rock porosity if the coated sample displaces 15.7 ml of water when fully submerged. \( \rho_g = 2.65 \text{ g/cm}^3 \), \( \rho_w = 1.00 \text{ g/cm}^3 \), \( \rho_v = 1.80 \text{ g/cm}^3 \).

\[
V_g = \frac{35.25}{2.65} = 13.30 \text{ cm}^3 \\
V_v = \frac{(36.55 - 35.25)}{1.80} = 0.72 \text{ cm}^3 \\
V_b = 15.70 - 0.72 = 14.98 \text{ cm}^3 \\
\phi = \frac{(14.98 - 13.3)}{14.98} = 0.112 = 11.2\%
\]

*Note:* This value is the absolute porosity.

Example 6
The varnish coating on the sample of Example 2.3 was removed and the sample was submerged in water. When air bubbling stopped, the sample was weighed while suspended in water. It weighed 21300 dynes. Assuming that you don't know the rock composition, compute the rock porosity.

Weight of sample in air = 35.25 x 980 = 34,545 dynes
Weight of sample in water = 21,300 dynes
Weight of displaced water = 34545 - 21300 = 13,245 dynes
Volume of displaced water = 13245 / (980 x 1) = 13.52 cm\(^3\)

\[
V_g = 13.52 \text{ cm}^3 \\
\phi = \frac{(14.98 - 13.52)}{14.98} = 0.097 = 9.7\%
\]

*Note:* This value is the effective porosity.
Example 7
A sandstone core plug is 1 inch in diameter, 2 inches long, and has a mass of 56.6 grams. When completely saturated with water, its mass increases to 60.9 grams. Compute the rock porosity.

\[
\text{Mass of water in sample} = 60.9 - 56.6 = 4.3 \text{ grams}
\]
\[
\text{Volume of water} = \frac{4.3}{1} = 4.3 \text{ cm}^3
\]
\[
V_b = \pi (0.5 \times 2.54)^2 \times 2 \times 2.54 = 25.74 \text{ cm}^3
\]
\[
\phi = \frac{4.3}{25.74} = 16.7\%
\]

*Note:* The absolute porosity of the sample computed from its estimated grain volume is 17%.

Example 8
A sonic log measured travel time of 58 μs for a formation. If the formation is primarily limestone (46 μs) and contains oil only (190 μs), compute the rock porosity.

Rearrange Equ. 2.6

\[
\phi = \frac{\Delta t_{\log} - \Delta t_{\text{ma}}}{\Delta t_f - \Delta t_{\text{ma}}}
\]
\[
= \frac{58 - 46}{190 - 46} = 8.3\%
\]

*Note:* This value is the average absolute porosity of the formation.
Homework

1-A dry core sample weighing 450 gm was saturated with brine ($\rho_{\text{brine}} = 1.05$). The saturated weight is 475 gm. The cylindrical core sample is 12 cm long and 4 cm in diameter. Calculate the core porosity? What is the type of the calculated porosity?

2-Calculate the bulk density of a rock which has 20% porosity. The bulk mineralogy of the rock is 80% quartz, 15% calcite and 5% clay. The pore fluid is composed of 20% brine and 80% hydrocarbon. Density values for quartz, calcite, clay, brine and hydrocarbons are 2.65, 2.71, 2.75, 1.0, 0.7 gm/cc respectively.

3-What is the porosity of a rock which is composed of 70% quartz, 20% calcite and 10% clay and has a dry bulk density of 2.31 gm/cc? On saturating the core with brine of density 1.02 gm/cc the saturated density was determined as 2.48 gm/cc. Using this value calculate the porosity. Why does it differ from the value calculated using dry density? Assume no errors in measurements.

4. Suppose we pack spherical balls, all of the same size, in a cubic packing arrangement. What will the porosity of the medium be?

5. Suppose the balls of Exercise 1 have 1 mm diameter. How big are the largest balls that we can fit in between them? What will the new porosity be?

6. A core plug 1 inch in diameter and 2 inches long is placed in the chamber of a gas expansion porosimeter whose container is 20 cm$^3$ large. The initial and final pressure readings are 25 and 21 psi, respectively. Ignoring the dead volume of the apparatus, what is the porosity of the sample?

7. A dry and clean core sample 1 inch in diameter and 4 inches long weighs 120 grams. Mineral analysis shows that the grains are 80% (by volume) calcite and 20% anhydrite. Estimate the sample's porosity. ($\rho_{\text{ca}} = 2.71$ g/cm$^3$, $\rho_{\text{an}} = 2.98$ g/cm$^3$)

8. Suppose the core sample of Problem 4 was obtained from a formation which contains only water ($\rho_w = 1$ g/cm$^3$). In one location of the formation, the density log measured a bulk density of 2.48 g/cm$^3$, what is the formation's porosity at that location?
# Appendix A. CONVERSION FACTORS

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 darcy</td>
<td>$9.87 \times 10^{-9}$ cm²</td>
</tr>
<tr>
<td>1 atm</td>
<td>1.013 bar</td>
</tr>
<tr>
<td></td>
<td>101,300 Pascal</td>
</tr>
<tr>
<td></td>
<td>1,013,000 dyne/cm²</td>
</tr>
<tr>
<td></td>
<td>14.7 psi</td>
</tr>
<tr>
<td>1 barrel (bbl)</td>
<td>5.615 ft³</td>
</tr>
<tr>
<td></td>
<td>42 gallons</td>
</tr>
<tr>
<td></td>
<td>159 liters</td>
</tr>
<tr>
<td></td>
<td>159,000 cm³</td>
</tr>
<tr>
<td></td>
<td>0.159 m³</td>
</tr>
<tr>
<td>1 pound (lb)</td>
<td>454 grams</td>
</tr>
<tr>
<td>1 short ton</td>
<td>2,000 lb</td>
</tr>
<tr>
<td>1 metric ton</td>
<td>1,000 kg</td>
</tr>
<tr>
<td></td>
<td>2,203 lb</td>
</tr>
<tr>
<td>1 foot (ft)</td>
<td>12 inches</td>
</tr>
<tr>
<td></td>
<td>30.5 cm</td>
</tr>
<tr>
<td>1 mile</td>
<td>5,280 ft</td>
</tr>
<tr>
<td></td>
<td>1,609 m</td>
</tr>
<tr>
<td>1 acre</td>
<td>43,560 ft²</td>
</tr>
<tr>
<td></td>
<td>4,047 m²</td>
</tr>
<tr>
<td>1 sq. mile</td>
<td>2,560,000 m²</td>
</tr>
<tr>
<td></td>
<td>640 acres</td>
</tr>
<tr>
<td>1 centipoise (cP)</td>
<td>1 mPa.s</td>
</tr>
<tr>
<td></td>
<td>0.01 g/cm.s</td>
</tr>
<tr>
<td></td>
<td>0.000672 lb/ft.s</td>
</tr>
</tbody>
</table>

$^0$K to $^0$C: $^0$C = $^0$K - 273.15
**Compressibility**

Knowledge of the variability of fluid compressibility with pressure and temperature is essential in performing many reservoir engineering calculations. For a liquid phase, the compressibility is small and usually assumed to be constant. For a gas phase, the compressibility is neither small nor constant.

- **Porosity is reduced by compaction**
  - Porosity reduction is determined by maximum burial depth
  - Principal effects are:
    - Changes in packing
    - Pressure solution
    - Recrystallization
    - Deformation of rock fragments

- **Compaction effects are not reversed by erosional unroofing (hysteresis effect)**

*Where the compressibility effect on the porosity.

**Mechanics of compaction**

[Diagram showing the mechanics of compaction with labels for Rotation and Closer Packing, Ductile Grain Deformation, Breakage of Brittle Grains, and Pressure Solution At Grain Contacts. Modified from Jonas and McBride, 1977.]
Fig. Relationship of original formation porosity to overburden $P_{ob}$ pressure. Where the overburden pressure due to deep under the surface (weigh of rocks) and pressure of fluids.

- **Importance**
  - Formation compressibility can have a significant impact on reservoir performance
  - Subsidence can have significant environmental impact

- **Types**
  - **Matrix Compressibility** ($C_m$): relative change in volume of solid rock material (grain volume) per unit pressure change (usually $C_m \equiv 0$).
  - **Pore Compressibility** ($C_p$): relative change in pore volume per unit pressure change.
  - **Bulk Compressibility** ($C_b$): relative change in bulk volume per unit pressure change (usually $\Delta V_b \equiv \Delta V_p$). Significant decrease in bulk volume can cause subsidence.
Quartz, for example, has a compressibility of about $2 \times 10^{-8}$ psi$^{-1}$ at 68 °F. Suppose a core sample is cut from a sandstone reservoir 6000 feet deep, where a reasonable $P_{ob}$ would be 6000 psia. If a quartz grain within the sample has a reservoir volume of 1 mm$^3$, its volume in the laboratory where $P_{ob}$ is atmospheric, would increase by

$$\Delta V = -c \cdot V \cdot \Delta P = -2 \times 10^{-8} \times 1 \times (14.7 - 6000) = 0.00012 \text{ mm}^3$$
Compressibility of Natural Gas

By definition, the isothermal gas compressibility is the change in volume per unit volume for a unit change in pressure or, in equation form:

\[ c_g = \frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T \]  

(2-44)

where \( c_g \) = isothermal gas compressibility, 1/psi.

From the real gas equation-of-state:

\[ V = \frac{nRTz}{P} \]

Where \( z \) gas deviation (compressibility) factor

\[ z = \frac{\text{actual } V \text{ of gas at } T, P}{\text{ideal } V \text{ of gas at } T, P} = \frac{V}{nRT/P} \]

Differentiating the above equation with respect to pressure at constant temperature \( T \) gives:

\[ \left( \frac{\partial V}{\partial P} \right)_T = nRT \left[ \frac{1}{p} \left( \frac{\partial z}{\partial P} \right) - \frac{z}{p^2} \right] \]

Substituting into Equation 2-44 produces the following generalized relationship:
It should be pointed out that Equation 2-46 is useful in determining the expected order of magnitude of the isothermal gas compressibility. Equation 2-45 can be conveniently (more accuracies) expressed in terms of the pseudo-reduced pressure and temperature by simply replacing \( p \) with \( (p_{pc} \cdot p_{pr}) \), or:

\[
p_{pr} = \frac{p}{p_{pc}}
\]

\[
T_{pr} = \frac{T}{T_{pc}}
\]

where

\( p = \) system pressure, psia
\( p_{pr} = \) pseudo-reduced pressure, dimensionless
\( T = \) system temperature, °R
\( T_{pr} = \) pseudo-reduced temperature, dimensionless
\( p_{pc}, T_{pc} = \) pseudo-critical pressure and temperature, respectively, and defined by the following relationships:

\[
p_{pc} = \sum_{i=1} y_i \cdot p_{ci}
\]

\[
T_{pc} = \sum_{i=1} y_i \cdot T_{ci}
\]

\[
\begin{align*}
\frac{1}{p_{pr} \cdot p_{pc}} - \frac{1}{Z \left[ \frac{\partial Z}{\partial (p_{pr} \cdot p_{pc})} \right]_{T_{pr}}} 
\end{align*}
\]

Multiplying the above equation by \( p_{pc} \) yields:
\[ c_g \, \frac{p_{pc}}{p_{pr}} = c_{pr} = \frac{1}{p_{pr}} - \frac{1}{Z} \left[ \frac{\partial Z}{\partial p_{pr}} \right]_{T_{pr}} \]  

(2-47)

The term \( c_{pr} \) is called the isothermal pseudo-reduced compressibility and is defined by the relationship:

\[ c_{pr} = c_g \, \frac{p_{pc}}{p_{pr}} \]  

(2-48)

where \( c_{pr} \) = isothermal pseudo-reduced compressibility
\( c_g \) = isothermal gas compressibility, psi\(^{-1}\)
\( p_{pc} \) = pseudo-reduced pressure, psi

Values of \( (\partial Z/\partial p_{pr})_{T_{pr}} \) can be calculated from the slope of the \( T_{pr} \) isotherm on the Standing and Katz \( z \)-factor chart.

Fig. Standing and Katz compressibility factors chart. (Courtesy of GPSA and GPA Engineering Data Book, EO Edition, 1987.)
Fig. Trube's pseudo-reduced compressibility for natural gas. (continuous)
**Oil compressibility**

Isothermal compressibility coefficients are required in solving many reservoir engineering problems, including transient fluid flow problems, and they are also required in the determination of the physical properties of the undersaturated crude oil.

By definition, the isothermal compressibility of a substance is defined mathematically by the following expression:

\[ c = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T \]

For a crude oil system, the isothermal compressibility coefficient of the oil phase \( c_o \) is defined for pressures above the bubble-point by one of the following equivalent expressions:

\[ c_o = -(1/V)(\partial V/\partial p)_T \]  
(2-93)

\[ c_o = -(1/B_o)(\partial B_o/\partial p)_T \]  
(2-94)

\[ c_o = (1/\rho_o)(\partial \rho_o/\partial p)_T \]  
(2-95)

where \( c_o \) = isothermal compressibility, \( \text{psi}^{-1} \)
\( \rho_o \) = oil density \( \text{lb/ft}^3 \)
\( B_o \) = oil formation volume factor, \( \text{bbl/STB} \)

At pressures below the bubble-point pressure, the oil compressibility is defined as:

\[ c_o = \frac{-1}{B_o} \left( \frac{\partial B_o}{\partial p} \right) + \frac{B_g}{B_o} \left( \frac{\partial R_s}{\partial p} \right) \]  
(2-96)

where \( B_g \) = gas formation volume factor, \( \text{bbl/scf} \)

\( B_g \) : The gas formation volume factor is used to relate the volume of gas, as measured at reservoir conditions, to the volume of the gas as measured at standard conditions, i.e., 60°F and 14.7 psia
There are several correlations that are developed to estimate the oil compressibility at pressures above the bubble-point pressure, i.e., undersaturated crude oil system. Three of these correlations are presented below:

- The Vasquez-Beggs correlation
- The Petrosky-Farshad correlation
- McCain’s correlation

**Vasquez-Beggs**

\[
c_o = \frac{-1,433 + 5R_{sb} + 17.2(T - 460) - 1,180 \gamma_{gs} + 12.61^\circ \text{API}}{10^5 p}
\]

where

- \( T \) = temperature, \( ^\circ \text{R} \)
- \( p \) = pressure above the bubble-point pressure, psia
- \( R_{sb} \) = gas solubility at the bubble-point pressure
- \( \gamma_{gs} \) = corrected gas gravity as defined by Equation 2-72

Note: The gas solubility \( R_s \) is defined as the number of standard cubic feet of gas that will dissolve in one stock-tank barrel of crude oil at certain pressure and temperature. The solubility of a natural gas in a crude oil is a strong function of the pressure, temperature, API gravity, and gas gravity. Glaso (1980) proposed a correlation for estimating the gas solubility as a function of the API gravity, pressure, temperature, and gas specific gravity. The proposed relationship has the following form:

\[
R_s = \gamma_g \left[ \left( \frac{\text{API}^{0.989}}{(T - 460)^{0.172}} \right) (p_b^*) \right]^{1.2255}
\]

where \( p_b^* \) is a correlating number and is defined by the following expression:

\[
p_b^* = 10^x
\]

with

\[
x = 2.8869 - [14.1811 - 3.3093 \log (p)]^{0.5}
\]
Petrosky-Farshad

\[ c_p = 1.705 \times 10^{-7} R_{sb}^{0.69357} \gamma_g^{0.1885} \text{API}^{0.3272} (T - 460)^{0.6729} p^{-0.5906} \]

where \( T = \) temperature, \( ^\circ \text{R} \)
\( R_{sb} = \) gas solubility at the bubble-point pressure, scf/STB

**Fatt's correlation**

He proposed empirical correlation that predict variation of porosity with pressure. In his correlation, Fatt defined the net overburden pressure, \( P_{ob,net} \), as

\[ P_{ob,net} = P_{ob} - 0.85 P_p \]

Where: \( P_{ob} \): overburden pressure, \( P_p \): pore pressure.

and the pore volume compressibility as

\[ c_p = - \frac{1}{V_p} \left( \frac{\partial V_p}{\partial P_p} \right)_T \]

Assuming that changes in \( V_b \) are small compared with changes in \( V_p \), the above equation can be rewritten as

\[ c_p = - \frac{1}{\varphi} \left( \frac{\partial \varphi}{\partial P_{ob,net}} \right)_T \]

Therefore, once \( c_p \) is estimated from the curves, the above equation can be used to estimate reduction in porosity from lab to reservoir as follows

\[ \Delta \varphi = - c_p \varphi_{lab} \Delta P_{ob,net} \]
Example 9

A hydrocarbon gas mixture has a specific gravity of 0.72. Calculate the isothermal gas compressibility coefficient at 2,000 psia and 140°F by assuming:

a. An ideal gas behavior
b. A real gas behavior

Solution

a. Assuming an ideal gas behavior, determine \( c_g \) by applying Equation 2-45:

\[
c_g = \frac{1}{2000} = 500 \times 10^{-6} \text{ psi}^{-1}
\]

b. Assuming a real gas behavior

Step 1. Calculate \( T_{pc} \) and \( p_{pc} \) by applying Equations 2-17 and 2-18

\[
T_{pc} = 168 + 325 \ (0.72) - 12.5 \ (0.72)^2 = 395.5 \ ^\circ\text{R}
\]
\[
p_{pc} = 677 + 15 \ (0.72) - 37.5 \ (0.72)^2 = 668.4 \ \text{psia}
\]

Where

**Case 1: Natural Gas Systems**

\[
T_{pc} = 168 + 325 \ \gamma_g - 12.5 \ \gamma_g^2
\]
\[
p_{pc} = 677 + 15.0 \ \gamma_g - 37.5 \ \gamma_g^2
\]

**Case 2: Gas-Condensate Systems**

\[
T_{pc} = 187 + 330 \ \gamma_g - 71.5 \ \gamma_g^2
\]

Step 2. Compute \( p_{pr} \) and \( T_{pr} \) from Equations 2-11 and 2-12.

\[
p_{pr} = \frac{2000}{668.4} = 2.99
\]
\[
T_{pr} = \frac{600}{395.5} = 1.52
\]
Step 3. Determine the z-factor from Figure 2-1:

\[ z = 0.78 \]

Step 4. Calculate the slope \([\partial z/\partial p_{pr}]_{T_{pr}} = 1.52\):

\[
\left[ \frac{\partial z}{\partial p_{pr}} \right]_{T_{pr}} = -0.022
\]

Step 5. Solve for \(c_{pr}\) by applying Equation 2-47:

\[ c_{pr} = \frac{1}{2.99} \left( -\frac{1}{0.78} \right) [ -0.022 ] = 0.3627 \]

Step 6. Calculate \(c_g\) from Equation 2-48:

\[ c_g = \frac{0.327}{668.4} = 543 \times 10^{-6} \text{ psi}^{-1} \]

Example 10
Using Trube's generalized charts, rework above example.

Solution

Step 1. From Figure 2-3, find \(c_{pr}\):

\[ c_{pr} = 0.36 \]

Step 2. Solve for \(c_g\) by applying Equation 2-49:

\[ c_g = \frac{0.36}{668.4} = 539 \times 10^{-6} \text{ psi}^{-1} \]

Matter, Brar, and Aziz (1975) presented an analytical technique for calculating the isothermal gas compressibility. The authors expressed \(c_{pr}\) as a function of \(\partial p/\partial \rho_r\) rather than \(\partial p/\partial p_{pr}\).
Example 11
A core plug was obtained from a reservoir where the overburden pressure is 5000 psi and the pore pressure is 2200 psi. If the plug shows a porosity of 18% in the laboratory, what is its porosity under reservoir conditions? Assume the pore compressibility follows curve E in Fig. 3.1.

\[
P_{ob,net} = P_{ob} - 0.85 \ P_p \\
= 5000 - 0.85 \times 2200 \\
= 3130 \ psi
\]

From Fig. 3.1, \( c_p = 10 \times 10^{-6} \ \text{psi}^{-1} \)

\[
\Delta \varphi = -c_p \ \varphi_{lab} \ \Delta P_{ob,net} \\
= -10 \times 10^{-6} \times 0.18 \times 3130 \\
= 0.0056
\]

Therefore, porosity under reservoir conditions is

\[
\varphi = 0.18 - 0.0056 \\
= 0.1744 \text{ or } 17.44\%
\]

*Note: Under laboratory conditions, net overburden pressure is usually negligible.*

Fig. 3.1: Variation of pore-volume compressibility with net overburden pressure

(from Fatt\(^1\))
Example 12

A gas reservoir has the following gas composition: the initial reservoir pressure and temperature are 3,000 psia and 180°F, respectively.

<table>
<thead>
<tr>
<th>Component</th>
<th>$y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>0.02</td>
</tr>
<tr>
<td>N$_2$</td>
<td>0.01</td>
</tr>
<tr>
<td>C$_1$</td>
<td>0.85</td>
</tr>
<tr>
<td>C$_2$</td>
<td>0.04</td>
</tr>
<tr>
<td>C$_3$</td>
<td>0.03</td>
</tr>
<tr>
<td>i - C$_4$</td>
<td>0.03</td>
</tr>
<tr>
<td>n - C$_4$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Calculate the gas compressibility factor under initial reservoir conditions.

**Solution**

<table>
<thead>
<tr>
<th>Component</th>
<th>$y_i$</th>
<th>$T_{ci},^\circ{\text{R}}$</th>
<th>$y_i T_{ci}$</th>
<th>$p_{ci}$</th>
<th>$y_i p_{ci}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>0.02</td>
<td>547.91</td>
<td>10.96</td>
<td>1071</td>
<td>21.42</td>
</tr>
<tr>
<td>N$_2$</td>
<td>0.01</td>
<td>227.49</td>
<td>2.27</td>
<td>493.1</td>
<td>4.93</td>
</tr>
<tr>
<td>C$_1$</td>
<td>0.85</td>
<td>343.33</td>
<td>291.83</td>
<td>666.4</td>
<td>566.44</td>
</tr>
<tr>
<td>C$_2$</td>
<td>0.04</td>
<td>549.92</td>
<td>22.00</td>
<td>706.5</td>
<td>28.26</td>
</tr>
<tr>
<td>C$_3$</td>
<td>0.03</td>
<td>666.06</td>
<td>19.98</td>
<td>616.4</td>
<td>18.48</td>
</tr>
<tr>
<td>i - C$_4$</td>
<td>0.03</td>
<td>734.46</td>
<td>22.03</td>
<td>527.9</td>
<td>15.84</td>
</tr>
<tr>
<td>n - C$_4$</td>
<td>0.02</td>
<td>765.62</td>
<td>15.31</td>
<td>550.6</td>
<td>11.01</td>
</tr>
</tbody>
</table>

$T_{pc} = 383.38 \quad p_{pc} = 666.38$

*Step 1.* Determine the pseudo-critical pressure from Equation 2-14:

$p_{pc} = 666.18$

*Step 2.* Calculate the pseudo-critical temperature from Equation 2-15:

$T_{pc} = 383.38$

*Step 3.* Calculate the pseudo-reduced pressure and temperature by applying Equations 2-12 and 2-13, respectively:

$p_{pr} = \frac{3000}{666.38} \approx 4.50$

$T_{pr} = \frac{640}{383.38} \approx 1.67$
Step 4. Determine the z-factor from Figure 2-1, to give:

\[ z = 0.85 \]

Equation 2-11 can be written in terms of the apparent molecular weight \( M_a \) and the weight of the gas \( m \):

\[ pV = z \left( \frac{m}{M_a} \right) RT \]

Solving the above relationship for the gas specific volume and density, give:

\[ v = \frac{V}{m} = \frac{zRT}{pM_a} \quad (2-16) \]

\[ \rho_g = \frac{1}{v} = \frac{pM_a}{zRT} \quad (2-17) \]

where \( v \) = specific volume, \( \text{ft}^3/\text{lb} \)
\[ \rho_g \] = density, \( \text{lb}/\text{ft}^3 \)

**Example 13**

Using the data in Example 2-5 and assuming real gas behavior, calculate the density of the gas phase under initial reservoir conditions. Compare the results with that of ideal gas behavior.

**Solution**

<table>
<thead>
<tr>
<th>Component</th>
<th>( y_i )</th>
<th>( M_i )</th>
<th>( y_i \cdot M_i )</th>
<th>( T_{ci} \cdot R )</th>
<th>( y_i T_{ci} )</th>
<th>( P_{ci} )</th>
<th>( y_i P_{ci} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>0.02</td>
<td>44.01</td>
<td>0.88</td>
<td>547.91</td>
<td>10.96</td>
<td>1071</td>
<td>21.42</td>
</tr>
<tr>
<td>N₂</td>
<td>0.01</td>
<td>28.01</td>
<td>0.28</td>
<td>227.49</td>
<td>2.27</td>
<td>493.1</td>
<td>4.93</td>
</tr>
<tr>
<td>C₁</td>
<td>0.85</td>
<td>16.04</td>
<td>13.63</td>
<td>343.33</td>
<td>291.83</td>
<td>666.4</td>
<td>566.44</td>
</tr>
<tr>
<td>C₂</td>
<td>0.04</td>
<td>30.1</td>
<td>1.20</td>
<td>549.92</td>
<td>22.00</td>
<td>706.5</td>
<td>28.26</td>
</tr>
<tr>
<td>C₃</td>
<td>0.03</td>
<td>44.1</td>
<td>1.32</td>
<td>666.06</td>
<td>19.98</td>
<td>616.40</td>
<td>18.48</td>
</tr>
<tr>
<td>i - C₄</td>
<td>0.03</td>
<td>58.1</td>
<td>1.74</td>
<td>734.46</td>
<td>22.03</td>
<td>527.9</td>
<td>15.84</td>
</tr>
<tr>
<td>n - C₄</td>
<td>0.02</td>
<td>58.1</td>
<td>1.16</td>
<td>765.62</td>
<td>15.31</td>
<td>550.6</td>
<td>11.01</td>
</tr>
</tbody>
</table>

\[ M_a = 20.23 \quad T_{pc} = 383.38 \quad P_{pc} = 666.38 \]
Step 1. Calculate the apparent molecular weight from Equation 2-5:

\[ M_a = 20.23 \]

Step 2. Determine the pseudo-critical pressure from Equation 2-14:

\[ p_{pc} = 666.18 \]

Step 3. Calculate the pseudo-critical temperature from Equation 2-15:

\[ T_{pc} = 383.38 \]

Step 4. Calculate the pseudo-reduced pressure and temperature by applying Equations 2-12 and 2-13, respectively:

\[ p_{pr} = \frac{3000}{666.38} = 4.50 \]

\[ T_{pr} = \frac{640}{383.38} = 1.67 \]

Step 5. Determine the z-factor from Figure 2-1:

\[ z = 0.85 \]

Step 6. Calculate the density from Equation 2-17:

\[ \rho_g = \frac{(3000)(20.23)}{(0.85)(10.73)(640)} = 10.4 \text{ lb/ft}^3 \]

Step 7. Calculate the density of the gas assuming an ideal gas behavior from Equation 2-7:

\[ \rho_g = \frac{(3000)(20.23)}{(10.73)(640)} = 8.84 \text{ lb/ft}^3 \]
Exercises

1. With the bulk, pore and grain compressibilities defined as

\[ c_b = - \frac{1}{V_b} \frac{\partial V_b}{\partial P} \quad c_p = - \frac{1}{V_p} \frac{\partial V_p}{\partial P} \quad c_g = - \frac{1}{V_g} \frac{\partial V_g}{\partial P} \]

respectively, show that the definition of porosity implies

\[ c_b = (1 - \phi) c_g + \phi c_p \]

2. The data below was obtained from a compressibility experiment on a core sample whose bulk volume is 52.0 cm\(^3\) and initial pore volume is 8.82 cm\(^3\). The sample's pore space is initially filled with water. Estimate the sample's pore-volume compressibility at 1000 and 2000-psi net overburden pressures.

<table>
<thead>
<tr>
<th>(P_p) (psi)</th>
<th>14.7</th>
<th>50</th>
<th>100</th>
<th>100</th>
<th>500</th>
<th>500</th>
<th>1000</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{ob}) (psi)</td>
<td>500</td>
<td>750</td>
<td>1000</td>
<td>1500</td>
<td>2000</td>
<td>2500</td>
<td>3000</td>
<td>3500</td>
</tr>
<tr>
<td>(V_{wp}) (cm(^3))</td>
<td>0.222</td>
<td>0.328</td>
<td>0.361</td>
<td>0.452</td>
<td>0.470</td>
<td>0.481</td>
<td>0.487</td>
<td>0.494</td>
</tr>
</tbody>
</table>

\(V_{wp}\) is total water squeezed out of sample at each step.

3. For a reservoir whose pore-volume compressibility is \(17 \times 10^{-6}\) psi\(^{-1}\), and where overburden pressure and pore pressure increase by 1 psi and 0.5 psi, respectively, per foot of depth, show that the porosity decreases by 1\% \((\phi_2/\phi_1 = 0.99)\) for every 1000 feet increase in depth.