Chapter 8: Flow of water through soils

8.1 Introduction

Among construction materials, soil is very unique. Because of a relatively large space of void in its constituent, water can flow through soil. The water flow (seepage) characteristics are very important in many applications of earthworks and structures such as earth dams, levees, embankments, underground structures, excavations, etc.

8.2 Soil moisture and modes of occurrence

Water present in the void spaces of a soil mass is called ‘Soil water’. Specifically, the term ‘soil moisture’ is used to denote that part of the sub-surface water which occupies the voids in the soil above the ground water table. Soil water may be in the forms of ‘free water’ or ‘gravitational water’ and ‘held water’, broadly speaking.

✧ To understand the system of flow of water throw soils , we much be acquainted with the following :

- All soils in natural are permeable materials, water being free to flow through the interconnected pores between the solid particles.

- The pressure of the pore water is one of the key parameters governing the strength and stiffness of soils.

- When pore water flow is occurring (this is known as seepage) and flow of water is soil is due to changing in head.

- The pressure of the pore water is measured relative to atmospheric pressure, and the level at which the pressure is atmospheric (i.e. zero) is defined as the water table (WT) or the phreatic surface.

- The level of the water table (Aquifer) changes according to climatic conditions, but the level can change also as a consequence of constructional operations.

- A perched water table can occur locally in an aquitard (in which water is contained by soil of low permeability, above the normal water table level) or an aquiclude (where the surrounding material is impermeable).
8.3 Bernoulli’s Equation

From fluid mechanics, we know that, according to Bernoulli’s equation, the total head at a point in water under motion can be given by the sum of the pressure, velocity, and elevation heads.

For soil mechanics, Bernoulli’s equation will be taken in term of heads:

\[ h_t = h_p + h_v + h_z \]

where

- \( h_t \): total head
- \( h_z \): elevation head
- \( h_p \): pressure head
- \( h_v \): velocity head
- \( u \): pore water pressure
- \( v \): flow velocity

The velocity head term \( v^2/2g \) is neglected in most soil mechanics problems since this value is quite small in comparison with the values of other terms, and thus

\[ h_t = h_p + h_z = \frac{u}{\gamma_w} + Z \]

It is very important to define the **datum** to use Equation above. **The datum can be chosen at any elevation, and all the heads are defined relative to the datum.**

As seen in Figure 8.1, \( z = h_z \) is the height at that point from the datum, and \( u/\gamma_w = h_p \) is the height of water in a standpipe with water pressure \( u \). The total head, \( h_t \), is the level of water in standpipes relative to the datum, and it constitutes a variety of combinations of \( h_z \) and \( h_p \). The values of the parameters appear in Figure 8.1 and are summarized in Table 8.1.
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FIGURE 8.1 Water flow through a pipe.

<table>
<thead>
<tr>
<th>Point</th>
<th>$h_z$</th>
<th>$h_p$</th>
<th>$h_t = h_z + h_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$Z_A$</td>
<td>0</td>
<td>$Z_A$ (= $h_t$ at B)</td>
</tr>
<tr>
<td>B</td>
<td>$Z_B$</td>
<td>$U_B/\gamma_w$</td>
<td>$Z_B + U_B/\gamma_w$ (= $h_t$ at A)</td>
</tr>
<tr>
<td>C</td>
<td>$Z_C$</td>
<td>$U_C/\gamma_w$</td>
<td>$Z_C + U_C/\gamma_w$</td>
</tr>
<tr>
<td>D</td>
<td>$Z_D$</td>
<td>$U_D/\gamma_w$</td>
<td>$Z_D + U_D/\gamma_w$ (= $h_t$ at E)</td>
</tr>
<tr>
<td>E</td>
<td>$Z_E$</td>
<td>0</td>
<td>$Z_E$ (= $h_t$ at D)</td>
</tr>
</tbody>
</table>

Table 8.1 demonstrates that $h_t$ values are the same at Points A and B and at Points D and E, although $h_z$ and $h_p$ are different at all the points. If there are no changes in $h_t$, it implies “no (total) head loss.” As seen in Figure 8.1 and Table 8.1, the only head loss occurs from Point B to Point D, where water flows through the soil.

Head loss is an energy loss. When water flows in soils, it must flow through many small passages in void sections of soils, as illustrated in Figure 8.2. This creates frictional resistance on the surfaces of particles. Flow energy is transmitted to frictional resistance on particle surfaces and then may be lost in heat generation, although it may not be easy to measure the temperature rise due to this energy transfer.
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8.3 Darcy’s Equation

The energy of water flow comes from the total head loss as described in the previous section, and it follows Darcy’s law in Equation below:

\[ v = k i \]
\[ q = v A = k i t A = k (\Delta h/L)A \]
\[ Q = q t = k i t A t = (k \Delta h A t)/L \]

where:
- \( v \): discharge velocity of water flow through porous media (m/s)
- \( k \): coefficient of permeability (m/s)
- \( i \): hydraulic gradient in a nondimensional form (head loss/flow length = \( \Delta h/L \))
- \( A \): cross-sectional area of specimen (pore spaces only) perpendicular to flow direction (m²)
- \( q \): flow rate of water (m³/s)
- \( Q \): total amount of flow (m³) for a period \( t \) (second)

Note that the discharge velocity \( v \) (or simply, velocity) in Equation \( v = k i \) is not the true velocity of water flow, but is rather an average velocity in the flow direction through the porous media, pore spaces only and not thought the entire cross-sectional area of specimen.
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Since water can flow only in the void section of the media, the true velocity of water (still in average in the direction of an average flow direction) must be faster than \( v \) to carry the same quantity of water.

The true velocity through the void is called seepage velocity \( v_s \) and is computed as

\[
q = vA = A_v v_s
\]

Where

\( v_s \) = seepage velocity
\( A_v \) = area of void in the cross section of the specimen
\( A = A_v + A_s \)

\[
q = v(A_v + A_s) = A_v v_s
\]

\[
v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \frac{v(V_v + V_s)}{V_v}
\]

where

\( V_v \) = volume of voids in the specimen
\( V_s \) = volume of soil solids in the specimen

\[
v_s = v \left[ \frac{1 + \frac{V_v}{V_s}}{\frac{V_v}{V_s}} \right] = v \left[ \frac{1+e}{e} \right] = \frac{v}{n}
\]

where \( n \) is the porosity of soils, in which area water can only flow relative to gross cross-sectional area 1 for discharge velocity \( v \). The real velocity of water molecules is even faster than \( v_s \) since real water passages are not straight but
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rather meandering with longer passages around the particles. The discharge velocity \( v \), however, has an engineering significance since it is a gross measure of velocity for a cross section of soils in an average flow direction. Discharge velocity is simply termed as velocity and is used in the following discussions.

Exercise 8.1

Figure below shows water flow though the soil specimen in a cylinder. The specimen’s \( k \) value is \( 3.4 \times 10^{-4} \) cm/s.

(a) Calculate pressure heads \( h_p \) at Points A, B, C, and D and draw the levels of water height in standpipes.

(b) Compute the amount of water flow \( q \) through the specimen.

In the \( h_t \) computation in the table, the head loss from A to B is one-third of the total head loss (80 mm). The same total head loss occurs from B to C and from C to D. The heights of water in standpipes are plotted in Figure.

<table>
<thead>
<tr>
<th>Point</th>
<th>( h_z ) (mm)</th>
<th>( h_t ) (mm)</th>
<th>( hp = ht - hz ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>280</td>
<td>280-50=230</td>
</tr>
<tr>
<td>B</td>
<td>75</td>
<td>280–80/3 = 253.3</td>
<td>253.3-75=178.3</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>253.3–80/3=226.6</td>
<td>226.6-100=126.6</td>
</tr>
<tr>
<td>D</td>
<td>125</td>
<td>226.6–80/3 = 200</td>
<td>200-125=75</td>
</tr>
</tbody>
</table>

(b) \( q = k (\Delta h/L)A = 3.4 \times 10^{-4} \times (8/18) \times \pi(6/2)2 = 4.27 \times 10^{-3} \) cm³/s

Asst.L. Musaab. S.A
8.4 Hydraulic Conductivity $k$:

Hydraulic conductivity is generally expressed in cm/sec or m/sec in SI units.

The hydraulic conductivity of soils depends on several factors:

1. fluid viscosity, $\eta$
2. pore size distribution,
3. grain-size distribution,
4. void ratio, $e$
5. roughness of mineral particles,
6. degree of soil saturation.

$$k = \frac{\gamma_w}{\eta} K$$

where $\gamma_w$ = unit weight of water  
$\eta$= viscosity of water  
$K$= absolute permeability

In clayey soils, structure plays an important role in hydraulic conductivity. Other major factors that affect the permeability of clays are the ionic concentration and the thickness of layers of water held to the clay particles.

The value of $k$ changes in a logarithmic way. For example, the $k$ value is more than $1 \times 10^{-1}$ cm/s for gravels, and it is less than $1 \times 10^{-7}$ cm/s for clayey soils.
8.5 Laboratory Determination of Hydraulic Conductivity $k$

There are two laboratory methods available: constant head permeability test (ASTM D 2434) and falling head permeability test.

8.5.1 Constant Head Permeability Test

As seen in Figure 8.3, the soil specimen is prepared in a vertical standing cylindrical mold, and a constant hydraulic head is applied. Under a steady-state flow condition, discharged water at the exit is collected in a cylinder as $Q$ for a time period $t$. From Darcy's Equation, $k$ is computed as

$$k = \frac{QL}{A \Delta h t}$$

FIGURE 8.3 Constant head permeability test.
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Where:

Q: collected amount of water for a time period t
L: length of soil specimen in the flow direction
A: cross-sectional area of a soil specimen
Δh: hydraulic head loss in constant head test setup

An average value from several trials is reported as the measured k value.

8.5.2 Falling Head Permeability Test

Figure 8.4 shows a setup for this test. The specimen is prepared similarly as in the constant head test. The higher head is applied through a burette in which the head changes with time. The head at the discharging side is constant as seen. At initial time (t = t₁), head loss is Δh₁, and at t = t₂, head loss is Δh₂. The amount of water flow “q” (per unit time) is equal to the change in head loss (dΔh) multiplied by the burette’s cross-sectional area “a” per time “dt.” Thus,

\[ q = -a \frac{dΔh}{dt} = k \frac{Δh}{L} A \]

\[ dt = \frac{aL}{Ak} \left[ \frac{-dΔh}{Δh} \right] \]

Integrate from t₁ to t₂, and for the corresponding Δh₁ to Δh₂:

\[ \int_{t_1}^{t_2} dt = (t_2 - t_1) = \frac{aL}{Ak} \int_{Δh_1}^{Δh_2} \left[ \frac{-dΔh}{Δh} \right] \]

\[ (t_2 - t_1) = \frac{aL}{Ak} ln \frac{Δh_1}{Δh_2} \]

\[ k = \frac{aL}{(t_2-t_1)A} ln \frac{Δh_1}{Δh_2} \]

The constant head test is usually used for coarse-grained soils and the falling head test for finer soils for rather accurate measurements with an effective use of testing time.
Example 8.2: Refer to the constant-head permeability test arrangement shown in Figure (8.3). A test gives these values:

- L = 30 cm
- A = area of the specimen = 177 cm²
- Constant-head difference, h = 50 cm
- Water collected in a period of 5 min = 350 cm³

Calculate the hydraulic conductivity in cm/sec.

Solution:

\[
k = \frac{QL}{Aht}
\]

Given Q = 350 cm³, L = 30 cm, A = 177 cm², h = 50 cm, and t = 5 min, we have;

\[
k = \frac{(350)(30)}{(177)(50)(5)(60)} = 3.95 \times 10^{-3} \text{ cm/sec}
\]
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Example 8.3
For a falling-head permeability test, the following values are given:

- Length of specimen = 20 cm
- Area of soil specimen = 10 cm²
- Area of standpipe = 0.4 cm²
- Head difference at time $t=0 = 50$ cm
- Head difference at time $t=180\text{sec} = 30$ cm

Determine the hydraulic conductivity of the soil in cm/sec.

Solution:

$$k = 2.303 \frac{aL}{At} \log_{10} \frac{\Delta h_1}{\Delta h_2}$$

We are given $a= 0.4 \text{ cm}^2$, $L= 20 \text{ cm}$, $A= 10 \text{ cm}^2$, $t= 180\text{sec}$, $h_1= 50 \text{ cm}$ and $h_2= 30 \text{ cm}$

$$k = 2.303 \frac{(0.4)(20)}{(10)(180)} \log_{10} \left(\frac{50}{30}\right) = 2.27 \times 10^{-3} \text{ cm/sec}$$

The laboratory permeability test is rather simple and is a cost-effective way to determine $k$ values. However, it should be realized that samples are reconstituted mostly for sand and gravels and, for cohesive soils, some degree of disturbance cannot be avoided during a sampling process, transportation to the laboratory, and insertion into the test mold. In particular, a specimen should be perfectly fitted into the inside of the mold to avoid any water flow through possible spaces between the inner wall of the mold and the specimen itself. For this reason,

8.6 Relationship between $i$ & $\nu$

$$i = \frac{\Delta h}{L}$$

Asst.L.Musaab .S.A
where

\( i = \) hydraulic gradient

\( L = \) distance between points \( A \) and \( B \)—that is, the length of flow over which the loss of head occurred

In general, the variation of the velocity \( v \) with the hydraulic gradient \( i \) is as shown in Figure 8.6. This figure is divided into three zones:

1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)

When the hydraulic gradient is increased gradually, the flow remains laminar in Zones I and II, and the velocity, \( v \), bears a linear relationship to the hydraulic gradient. At a higher hydraulic gradient, the flow becomes turbulent (Zone III).

When the hydraulic gradient is decreased, laminar flow conditions exist only in Zone I. In most soils, the flow of water through the void spaces can be considered laminar; thus,

\[ v \propto i \]
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![Image of velocity vs hydraulic gradient]

**Figure 8.6** Nature of variation of $v$ with hydraulic gradient, $i$

**Example 8.4**

Find the flow rate in m$^3$/sec/m length (at right angle to the cross section shown) through the permeable soil layer shown in Figure below given $H = 8$ m, $H_1 = 3$ m, $h = 4$ m, $L = 50$ m, $\alpha = 8^\circ$, and $k = 0.08$ cm/sec.
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Solution:

Hydraulic gradient \( (i) = \frac{h}{L \cos \alpha} \)

\[ q = k_i A = (k) \left( \frac{h \cos \alpha}{L} \right) (H_1 \cos \alpha \times 1) \]

\[ q = (0.08 \times 10^{-2} \text{ m/sec}) \left( \frac{A \cos 8}{50} \right) (3 \cos 8 \times 1) = 0.19 \times 10^{-3} \text{ m}^3/\text{sec/m} \]

8.7 Directional Variation of Permeability

Most soils are not isotropic with respect to permeability. In a given soil deposit, the magnitude of \( k \) changes with respect to the direction of flow. Figure 8.7 shows a soil layer through which water flows in a direction inclined at an angle \( \alpha \) with the vertical.

Figure 8.7 Directional variation of permeability

Let the hydraulic conductivity in the vertical (\( \alpha = 0 \)) and horizontal (\( \alpha = 90^\circ \)) directions be \( k_V \) and \( k_H \), respectively. The magnitudes of \( k_V \) and \( k_H \) in a given soil depend on several factors, including the method of deposition in the field.

The laboratory test results obtained by Fukushima and Ishii (1986) related to \( k_V \) and \( k_H \) for compacted Masa-do soil shows the Variation of \( k_V \) and \( k_H \), (with respect to the direction of flow in the other words)
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8.8 Equivalent Hydraulic Conductivity in Stratified Soil

In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations.

8.8.1 Flow in the Horizontal Direction

Figure (8.8) shows n layers of soil with flow in the horizontal direction. Let us consider a cross section of unit length passing through the n layers and perpendicular to the direction of flow. The total flow through the cross section in unit time can be written as

\[ q = v \cdot 1. H = v_1 \cdot 1. H_1 + v_2 \cdot 1. H_2 + v_3 \cdot 1. H_3 + \cdots + v_n \cdot 1. H_n \]

Where:
- \( v \) = average discharge velocity
- \( v_1, v_2, v_3, \ldots, v_n \) = discharge velocities of flow in layers denoted by the subscripts

If \( k_{H1}, k_{H2}, k_{H3}, \ldots, k_{Hn} \) are the hydraulic conductivities of the individual layers in the horizontal direction and \( k_{H(eq)} \) is the equivalent hydraulic conductivity in the horizontal direction, then, from Darcy’s law

\[ v = k_{H(eq)} i_{eq}; \ v_1 = k_{H1} i_1; \ v_2 = k_{H2} i_2; \ v_3 = k_{H3} i_3; \ldots \ v_n = k_{Hn} i_n \]

Noting that \( i_{eq} = i_1 = i_2 = i_3 = \ldots = i_n \)

\[ k_{H(eq)} = \frac{1}{H} (k_{H1} H_1 + k_{H2} H_2 + k_{H3} H_3 + \cdots + k_{Hn} H_n) \]

Figure 8.8 Equivalent hydraulic conductivity determination—horizontal flow in stratified soil
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8.8.2 Flow in the Vertical Direction

Figure 8.9 shows $n$ layers of soil with flow in the vertical direction. In this case, the velocity of flow through all the layers is the same. However, the total head loss, $h$, is equal to the sum of the head losses in all layers. Thus,

$$\nu = \nu_1 = \nu_1 = \cdots = \nu_n$$

and

$$h = h_1 + h_2 + h_3 + \cdots + h_n$$

Using Darcy’s law, we can rewrite

$$k_{v(eq)} \left( \frac{h}{H} \right) = k_{v_1} i_1 = k_{v_2} i_1 = k_{v_3} i_3 = \cdots = k_{v_n} i_n \quad \text{-------(1)}$$

where: $k_{v_1}, k_{v_2}, k_{v_3}, \ldots, k_{v_n}$ are the hydraulic conductivities of the individual layers in the vertical direction and $k_{v(eq)}$ is the equivalent hydraulic conductivity.

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n \quad \text{------------------------(2)}$$

Solving Eqs. (1) and (2) gives

$$k_{v(eq)} = \frac{H}{\left( \frac{H_1}{k_{v_1}} \right) + \left( \frac{H_2}{k_{v_2}} \right) + \left( \frac{H_3}{k_{v_3}} \right) + \cdots + \left( \frac{H_n}{k_{v_n}} \right)}$$
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Example 8.5

A layered soil is shown in Figure below, Given:

\[ H_1 = 1.5 \text{ m} \quad k_1 = 1 \times 10^{-4} \text{ cm/sec} \]
\[ H_2 = 3 \text{ m} \quad k_2 = 3.2 \times 10^{-2} \text{ cm/sec} \]
\[ H_3 = 2 \text{ m} \quad k_3 = 4.1 \times 10^{-5} \text{ cm/sec} \]

Estimate the ratio of equivalent hydraulic conductivity, \( k_{H(eq)} \) and \( k_{v(eq)} \)

\[
k_{H(eq)} = \frac{1}{H} \left( k_{H_1} H_1 + k_{H_2} H_2 + k_{H_3} H_3 \right)
\]

\[
k_{H(eq)} = \frac{1}{1.5 + 3 + 2} \left( (1.5 \times 10^{-4}) + (3 \times 3.2 \times 10^{-2}) + (2 \times 4.1 \times 10^{-5}) \right) = 148.05 \times 10^{-4}
\]

\[
k_{v(eq)} = \frac{H}{\left( \frac{H_1}{k_{v_1}} + \frac{H_2}{k_{v_2}} + \frac{H_3}{k_{v_3}} \right)}
\]

\[
k_{v(eq)} = \frac{1.5 + 3 + 2}{\left( \frac{1.5}{10^{-4}} \right) + \left( \frac{3}{3.2 \times 10^{-2}} \right) + \left( \frac{2}{4.1 \times 10^{-5}} \right)} = 1.018 \times 10^{-4} \text{ cm/sec}
\]

\[
\frac{k_{H(eq)}}{k_{v(eq)}} = \frac{148.05 \times 10^{-4}}{1.018 \times 10^{-4}} = 145.4
\]
Example 8.6

Figure below shows three layers of soil in a tube that is 100 mm × 100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

<table>
<thead>
<tr>
<th>Soil</th>
<th>k(cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>B</td>
<td>$3 \times 10^{-3}$</td>
</tr>
<tr>
<td>C</td>
<td>$4.9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Find the rate of water supply in cm$^3$/hr

\[
q = k_{v(eq)}^{-1} A = (0.001213 \left( \frac{300}{450} \right) \left( \frac{100}{10} \times \frac{100}{10} \right))
\]

\[
= 0.0809 \text{ cm}^3/\text{sec} = 291.24 \text{ cm}^3/\text{hr}
\]
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8.9 Field determination of coefficient of permeability

An alternative way to obtain a more representative and reliable k value is to use field methods, although these may be relatively expensive. The classic field permeability test methods involve pumping water from a well and observing water table changes in two observation wells.

8.9.1 Unconfined Permeable Layer Underlain by Impervious Layer (Unconfined Aquifer)

As seen in Figure 8.10, a well is excavated through the permeable layer, and two observation wells are installed at \( r_1 \) and \( r_2 \) distances from the center of the center well hole. Water is pumped out at a steady rate until the height of the water level at the center well as well as at the two observation wells becomes stable. The depths of water in these observation wells are \( h_1 \) and \( h_2 \).

![Figure 8.10 Field permeability test for unconfined permeable layer underlain by the impervious layer.](image)

Let \( h \) be the depth of water at radial distance \( r \). The area of the vertical cylindrical surface of radius \( r \) and depth \( h \) through which water flows is

\[
A = 2\pi rh
\]
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hydraulic gradient is \[ i = \frac{dh}{dr} \]

As per Darcy's law the rate of inflow into the well when the water levels in the wells remain stationary is \[ q = kiA \]

Substituting for \( A \) and \( i \) the rate of inflow across the cylindrical surface

\[ q = k \frac{dh}{dr} 2\pi rh \]

Rearranging the terms, we have

\[ \frac{dr}{r} = \frac{2\pi h. dh}{q} \]

The integral of the equation within the boundary limits

\[ \int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2\pi h. dh}{q} \]

The equation for \( k \) after integration and rearranging is

\[ k = \frac{2.3q}{\pi(h_2^2 - h_1^2)} \log \frac{r_2}{r_1} \]

Where

- \( q \): amount of pumped water per unit time
- \( r_1 \) and \( r_2 \): distances of observation wells from the center of the center well hole
- \( h_1 \) and \( h_2 \): observed water heights at observation wells as defined in Figure 8.10

Proceeding in the same way as before another equation for \( k \) in terms of \( r_0 \), \( h_0 \) and \( R_i \) can be established as (referring to Fig.8.10)

\[ k = \frac{2.3q}{\pi(H^2 - h_0^2)} \log \frac{R_i}{r_0} \]

We write \( h_0 = (H-D_0) \), where \( D_0 \) is the depth of maximum drawdown in the test well, we have
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\[
k = \frac{2.3q}{\pi D_0(2H - D_0)} \log \frac{R_i}{r_0}
\]

**Radius of Influence** $R_i$ Based on experience, Sichardt (1930) gave an equation for estimating the radius of influence for the stabilized flow condition as

\[R_i = 3000D_0\sqrt{k}\] meters

where

$D_0$ = maximum drawdown in meters
$k$ = hydraulic conductivity in m/sec

**8.9.2 Pervious layer enclosed by two Impervious stratum (Confined Aquifer)**

The figure 8.11 shows a *confined aquifer* with the test and observation wells. The water in the observation wells rises above the top of the aquifer due to artesian observation wells rises above the top of the aquifer due to *artesian pressure*.

![Diagram of confined aquifer](image)

**Figure 8.11: Pumping test in confined aquifer**
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When pumping from such an artesian well two cases might arise. They are:

Case 1:
The water level in the test well might remain above the roof level of the aquifer at steady flow conditions.

Case 2:
The water level in the test well might fall below the roof level of the aquifer at steady flow conditions.

If $H_0$ is the thickness of the confined aquifer and $h_0$ is the depth of water in the test well at the steady flow condition, Case 1 and Case 2 may be stated as:

**Case 1. When** $h_0 > H_0$

In this case, the area of a vertical cylindrical surface of any radius $r$ does not change, since the depth of the water bearing strata is limited to the thickness $H_0$. Therefore, the discharge surface area:

$$A = 2\pi r H_0$$

hydraulic gradient is

$$i = \frac{dh}{dr}$$

$$q = k i A = k \frac{dh}{dr} 2\pi r H_0$$

$$\int_{h_1}^{h_2} dh = \frac{q}{k 2\pi H_0} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$(h_2 - h_1) = \frac{q}{k 2\pi H_0} \log \frac{r_2}{r_1}$$

$$k = \frac{2.3q}{2\pi H_0 (h_2 - h_1)} \log \frac{r_2}{r_1}$$
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Alternate Equation

$$k = \frac{2.3q}{2\pi H_0(h_1 - h_0)} \log \frac{r_2}{r_0}$$

or

$$k = \frac{2.3q}{2\pi H_0(H - h_0)} \log \frac{R_i}{r_0}$$

or

$$k = \frac{2.3q}{2\pi H_0D_0} \log \frac{R_i}{r_0}$$

Case 2. When $h_0<H_0$

Under the condition when $h_0$ is less than $H_0$, the flow pattern close to the well is similar to that unconfined aquifer whereas at distances farther from the well the flow is artesian. Muskat (1946) developed an equation to determine the hydraulic conductivity. The equation,

$$k = \frac{2.3q}{\pi(2HH_0 - H^2 - h_0^2)} \log \frac{R_i}{r_0}$$

Example 8.7

A pumping test was carried out for determining the hydraulic conductivity of soil in place. A well diameter 40 cm was drilled down to an impermeable stratum. The depth of water above the bearing stratum was 8 m. The yield from the well was 4 m$^3$/min at a steady drawdown of 4.5 m. Determine the hydraulic conductivity of the soil in m/day if the observed radius of influence was 150m

**solution**

$$k = \frac{2.3q}{\pi D_0(2H - D_0)} \log \frac{R_i}{r_0}$$

$q = 4 \text{ m}^3/\text{min} = 4 \times 60 \times 24 \text{ m}^3/\text{day}$, $D_0 = 4.5 \text{ m}$, $H = 8 \text{ m}$, $R_i = 150 \text{ m}$, $r_0 = 0.2 \text{ m}$

$$k = \frac{2.34 \times 60 \times 24}{3.14 \times 4.5(2 \times 8 - 4.5)} \log \frac{150}{0.2} = 234.4 \text{ m/day}$$
Example 8.8

A field pumping test was conducted from an aquifer of sandy soil of 4 m thickness confined between two impervious strata. When equilibrium was established, 90 liters of water was pumped out per hour. The water elevation in an observation well 3.0 m away from the test well was 2.1 m and another 6.0 m away was 2.7 m from the roof level of the impervious stratum of the aquifer. Find the value of \( k \) of the soil in m/sec.

\[
k = \frac{2.3q}{2\pi H_0 (h_2 - h_1)} \log \frac{r_2}{r_1}
\]

\[
q = 90 \times 10^{-3} \text{ cm}^3/\text{hr} = 25 \times 10^{-6} \text{ m}^3/\text{sec}
\]

\[
k = \frac{2.3 \times 25 \times 10^{-6}}{2 \times 3.14 \times 4(2.7 - 2.1)} \log \frac{6}{3} = 1.148 \times 10^{-6}
\]
8.10 Seepage of Water Through the Soil

‘Seepage’ is defined as the flow of a fluid, usually water, through a soil under a hydraulic gradient. A hydraulic gradient is supposed to exist between two points if there exists a difference in the ‘hydraulic head’ at the two points.

8.11 Learning Objective

넓게 Derive Laplace's equation for ground water flow in 3-D from Darcy's law and the continuity equation.
넓게 Describe the flow lines and equipotential lines and the relationship between them.
넓게 Draw the flow net for 2-D confined flow with simple boundary conditions (e.g. the flow under a sheet pile)

8.12 Laplace’s Equation of Continuity of 2-D Flow Through Soils

To derive the Laplace differential equation of continuity, let us consider a single row of sheet piles that have been driven into a permeable soil layer, as shown in Figure (8.12a). The row of sheet piles is assumed to be impervious.

Figure 8.12 (a) Single-row sheet piles driven into permeable layer
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The steady-state flow of water from the upstream to the downstream side through the permeable layer is a two-dimensional flow. For flow at a point $A$, we consider an elemental soil block. The block has dimensions $dx$, $dy$, and $dz$ (length $dy$ is perpendicular to the plane of the paper); it is shown in an enlarged scale in Figure (8.12b).

$$ q = v_A ; v = ki $$

![Diagram](image)

**Figure 8.12** (b) flow at $A$ block

Form Continuity Equation:

$$ \left( \text{(flow rate) horizontal and vertical direction} \right)_{\text{in}} = \left( \text{(flow rate) horizontal and vertical direction} \right)_{\text{out}} $$

<table>
<thead>
<tr>
<th>Direction</th>
<th>(flow rate (q) ) _in</th>
<th>(flow rate (q) ) _out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>$v_x ; dz ; dy$</td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>$v_z ; dx ; dy$</td>
<td></td>
</tr>
</tbody>
</table>
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Assuming that water is incompressible and that no volume change in the soil mass occurs, from continuity equation we know that the total rate of inflow should equal the total rate of outflow. Thus,

\[ [v_x dzdy + v_z dxdy] = \left[ \left( v_x + \frac{\partial v_x}{\partial x} \right) dx \right] dzdy + \left( v_z + \frac{\partial v_z}{\partial z} \right) dy \]

After simplification;

\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \]

From Darcy’s law, the discharge velocities can be expressed as

\[ v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \]

\[ v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \]

where \( k_x \) and \( k_z \) are the hydraulic conductivities in the horizontal and vertical directions, respectively.

\[ \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial z} v_z = 0 ; \]

\[ k_x \left( \frac{\partial}{\partial x} \right) \frac{\partial h}{\partial x} + k_z \left( \frac{\partial}{\partial z} \right) \frac{\partial h}{\partial z} = 0 \]

Then;

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \]

If the soil is isotropic with respect to the hydraulic conductivity that is, \( k_x = k_z \) the preceding continuity equation for two-dimensional flow simplifies to

\[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \]
8.13 Flow Net
The solution of the continuity equation is represented by a family of flow lines and a family of equipotentials, constituting what is referred to as a Flow net. A flow line is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium. An equipotential line is a line along which the potential head at all points is equal. Flow net sketching leads to a greater understanding of seepage principles.

Steps must be followed to draw the Flow Net:
1- Draw the problem with a specified scale.
2- Draw the flow lines which has the following properties:
   - The first flow line must be in touch with the structure.
   - The last flow line must be in touch with the impervious layer under the structure.
   - The flow lines must be not less than 5 including the first and last one.
   - The distance between the flow lines must be equal.
3- Draw the equipotential line which has the following properties:
   - First equipotential line must be in touch with the ground of the upstream.
   - Last equipotential line must be in touch with the ground of the downstream.
   - Equipotential line must be normal to the flow lines to generate the Field.