Generation of High D.C. and A.C. Voltages

There are various applications of high d.c voltages in industries, research medical sciences etc. HVDC transmission over both overhead lines and underground cables is becoming more and more popular. HVDC is used for testing HVAC cables of long lengths as these have very large capacitance and would require very large values of currents if tested on HVAC voltages. Even though D.C. tests on A.C. cables is convenient and economical, these suffer from the fact that the stress distribution within the insulating material is different from the normal operating condition. In industry it is being used for electrostatic precipitation of ashing in thermal power plants, electrostatic painting, cement industry, communication systems etc. HVDC is also being used extensively in physics for particle acceleration and in medical equipments (X-Rays). The most efficient method of generating high D.C. voltages is through the process of rectification employing voltage multiplier circuits. Electrostatic generators have also been used for generating high D.C. voltages.

3.1 HALF-WAVE RECTIFIER CIRCUIT

The simplest circuit for generation of high direct voltage is the half wave rectifier shown in Fig. 2.1 Here RL is the load resistance and C the capacitance to smoothen the d.c. output voltage.

If the capacitor is not connected, pulsating d.c. voltage is obtained at the output terminals whereas with the capacitance C, the pulsation at the output terminal are reduced. Assuming the ideal transformer and small internal resistance of the diode during conduction the capacitor C is charged to the maximum voltage $V_{max}$ during conduction of the diode D. Assuming that there is no load connected, the d.c. voltage across capacitance remains constant at $V_{max}$ whereas the supply voltage oscillates between $-V_{max}$ and during negative half cycle the potential of point A becomes $-V_{max}$ and hence the diode must be rated for $2V_{max}$. This would also be the case if the transformer is grounded at A instead of B as shown in Fig. 2.1 (a). Such a circuit is known as voltage doubler due to Villard for which the output voltage would be taken across D. This d.c. voltage, however, oscillates between zero and $2V_{max}$ and is needed for the Cascade circuit.
If the circuit is loaded, the output voltage does not remain constant at $V_{max}$. After point $E$ (Fig. 2.1 (c)), the supply voltage becomes less than the capacitor voltage, diode stops conducting. The capacitor can not discharge back into the a.c. system because of one way action of the diode. Instead, the current now flows out of $C$ to furnish the current $i_L$ through the load. While giving up this energy, the capacitor voltage also decreases at a rate depending on the time constant $CR$ of the circuit and it reaches the point $F$ corresponding to $V_{min}$. Beyond $F$, the supply voltage is greater than the capacitor voltage and hence the diode $D$ starts conducting charging the capacitor $C$ again to $V_{max}$ and also during this period it supplies current to the load also. This second pulse of $ip(ic + iL)$ is of shorter duration than the initial charging pulse as it serve mainly to restore into $C$ the energy that $C$ meanwhile had supplied to load. Thus, while each pulse of diode current lasts much less than a half cycle, the load receives current more continuously from $C$.

Assuming the charge supplied by the transformer to the load during the conduction period $t$, which is very small to be negligible, the charge supplied by the transformer to the capacitor during conduction equals the charge supplied by the capacitor to the load. Note that $ic >> iL$. During one period $T = 1/f$ of the a.c voltage, a charge $Q$ is transferred to the load $RL$ and is given as
Equation (2.5) shows that the ripple in a rectifier output depends upon the load current and the circuit parameter like $f$ and $C$. The product $fC$ is, therefore, an important design factor for the rectifiers. The higher the frequency of supply and larger the value of filtering capacitor the smaller will be the ripple in the d.c. output. The single phase half-wave rectifier circuits have the following disadvantages:

(i) The size of the circuits is very large if high and pure d.c. output voltages are desired.

(ii) The h.t. transformer may get saturated if the amplitude of direct current is comparable with the nominal alternating current of the transformer.

It is to be noted that all the circuits considered here are able to supply relatively low currents and therefore are not suitable for high current applications such as HVDC transmission. When high d.c. voltages are to be generated, voltage doubler or cascaded voltage multiplier circuits are used. One of the most popular doubler circuit due to Greinacher is shown in Fig. 2.2. Suppose $B$ is more positive with respect to $A$ and the diode $D1$ conducts thus charging the capacitor $C1$ to $V_{max}$ with polarity as shown in Fig. 2.2. During the next half cycle terminal $A$ of the capacitor $C1$ rises to $V_{max}$ and hence terminal $M$ attains a potential of $2V_{max}$. Thus, the capacitor $C2$ is charged to $2V_{max}$ through $D2$. Normally the voltage across the load will be less than $2V_{max}$ depending upon the time constant of the circuit $C2RL$. 

$$Q = \int_{t}^{r} i_L(t) \, dt = \int_{t}^{r} \frac{V_{RL}(t)}{R_L} \, dt = IT = \frac{I}{f}$$

where $I$ is the mean value of the d.c output $i_L(t)$ and $V_{RL}(t)$ the d.c. voltage which includes a ripple as shown in Fig. 2.1 (c).

This charge is supplied by the capacitor over the period $T$ when the voltage changes from $V_{max}$ to $V_{min}$ over approximately period $T$ neglecting the conduction period of the diode.

Suppose at any time the voltage of the capacitor is $V$ and it decreases by an amount of $dV$ over the time $dt$ then charge delivered by the capacitor during this time is

$$dQ = CdV$$

Therefore, if voltage changes from $V_{max}$ to $V_{min}$, the charge delivered by the capacitor

$$\int dQ = \int_{V_{max}}^{V_{min}} CdV = - C(V_{max} - V_{min})$$

Or the magnitude of charge delivered by the capacitor

$$Q = C(V_{max} - V_{min}) \quad (2.3)$$

Using equation (2.2)

$$Q = 2\delta VC \quad (2.4)$$

Therefore,

$$2\delta VC = IT$$

or

$$\delta V = \frac{IT}{2C} = \frac{I}{2fC} \quad (2.5)$$
3.2 COCKROFT-WALTON VOLTAGE MULTIPLIER CIRCUIT

In 1932, Cockroft and Walton suggested an improvement over the circuit developed by Greinacher for producing high D.C. voltages. Fig. 2.3. Shows a multistage single phase cascade circuit of the Cockroft-Walton type.

No Load Operation: The portion ABM'MA is exactly identical to Greinacher voltage doubler circuit and the voltage across C becomes 2Vmax when M attains a voltage 2Vmax. During the next half cycle when B becomes positive with respect to A, potential of M falls and, therefore, potential of N also falls becoming less than potential at M' hence C2 is charged through D2. Next half cycle A becomes more positive and potential of M and N rise thus charging C'2 through D'2. Finally all the capacitors C'1, C'2, C'3, C1, C2, and C3 are charged. The voltage across the column of capacitors consisting of C1, C2, C3, keeps on oscillating as the supply voltage alternates. This column, therefore, is known as oscillating column. However, the voltage across the capacitances C'1, C'2, C'3, remains constant and is known as smoothening column. The voltages at M', N', and O' are 2Vmax 4Vmax and 6Vmax. Therefore, voltage across all the capacitors is 2Vmax except for C1 where it is Vmax only. The total output voltage is 2nVmax where n is the number of stages.
use of multistages arranged in the manner shown enables very high voltage to be obtained. The equal stress of the elements (both capacitors and diodes) used is very helpful and promotes a modular design of such generators.

*Generator Loaded:* When the generator is loaded, the output voltage will never reach the value $2n V_{max}$. Also, the output wave will consist of ripples on the voltage. Thus, we have to deal with two quantities, the voltage drop $\Delta V$ and the ripple $\delta V$.

Suppose a charge $q$ is transferred to the load per cycle. This charge is $q = IIf = IT$.

The charge comes from the smoothening column, the series connection of $C'1, C'2, C'3$, If no charge were transferred during $T$ from this stack via $D1, D2, D3$, to the oscillating column, the peak to peak ripple would merely be
\[ 2\delta V = IT \sum_{n=0}^{n} \frac{1}{C_i} \]  

But in practice charges are transferred. The process is explained with the help of circuits in Fig. 2.4 (a) and (b).

Fig. 2.4 (a) shows arrangement when point A is more positive with reference to B and charging of smoothing column takes place and Fig. 2.4 (b) shows the arrangement when in the next half cycle B becomes positive with reference to A and charging of oscillating column takes place. Refer to Fig. 2.4 (a). Say the potential of point \( O' \) is now 6 \( V_{max} \). This discharges through the load resistance and say the charge lost is \( q = IT \) over the cycle. This must be regained during the charging cycle (Fig. 2.4 (a)) for stable operation of the generator. \( C_3 \) is, therefore supplied a charge \( q \) from \( C_3 \). For this \( C_2 \) must acquire a charge of 2\( q \) so that it can supply \( q \) charge to the load and \( q \) to \( C_3 \), in the next half cycle termed by cockroft and Walton as the transfer cycle (Fig. 2.4 (b)). Similarly \( C'_1 \) must acquire for stability reasons a charge 3\( q \) so that it can supply a charge \( q \) to the load and 2\( q \) to the capacitor \( C_2 \) in the next half cycle (transfer half cycle).
During the transfer cycle shown in Fig. 2.4 (b), the diodes \(D1, D2, D3\), conduct when \(B\) is positive with reference to \(A\). Here \(C'2\) transfers \(q\) charge to \(C3\), \(C1\) transfers charge \(2q\) to \(C2\) and the transformer provides change \(3q\).

For \(n\)-stage circuit, the total ripple will be

\[
2\delta V = \frac{I}{f} \left( \frac{1}{C'_{n}} + \frac{2}{C'_{n-1}} + \frac{3}{C'_{n-2}} + \cdots + \frac{n}{C'_{1}} \right)
\]

or

\[
\delta V = \frac{I}{2f} \left( \frac{1}{C'_{n}} + \frac{2}{C'_{n-1}} + \frac{3}{C'_{n-2}} + \cdots + \frac{n}{C'_{1}} \right)
\] (2.7)

From equation (2.7), it is clear that in a multistage circuit the lowest capacitors are responsible for most ripple and it is, therefore, desirable to increase the capacitance in the lower stages. However, this is objectionable from the view point of High Voltage Circuit where if the load is large and the load voltage goes down, the smaller capacitors (within the column) would be overstressed. Therefore, capacitors of equal value are used in practical circuits \(i.e.,\ C'n = C'n - 1 = \ldots C'1 = C\) and the ripple is given as

\[
\delta V = \frac{I}{2fC} \frac{n(n+1)}{2} = \frac{In(n+1)}{4fC}
\] (2.8)

The second quantity to be evaluated is the voltage drop \(\Delta V\) which is the difference between the theoretical no load voltage \(2nVmax\) and the on load voltage. In order to obtain the voltage drop \(\Delta V\) refer to Fig. 2.4 (a).

Here \(C'1\) is not charged upto full voltage \(2Vmax\) but only to \(2Vmax - 3q/C\) because of the charge given up through \(C1\) in one cycle which gives a voltage drop of \(3q/C = 3I/fC\).

The voltage drop in the transformer is assumed to be negligible. Thus, \(C2\) is charged to the voltage

\[
\left(2V_{max} - \frac{3I}{fC} \right) - \frac{3I}{fC}
\]

since the reduction in voltage across \(C'3\) again is \(3I/fC\). Therefore, \(C'2\) attains the voltage
Here again the lowest capacitors contribute most to the voltage drop $\Delta V$ and so it is advantageous to increase their capacitance in suitable steps. However, only a doubling of $C_1$ is convenient as this capacitor has to withstand only half of the voltage of other capacitors. Therefore, $\Delta V_1$ decreases by an amount $nI/fC$ which reduces $\Delta V$ of every stage by the same amount i.e., by

$$n \cdot \frac{nI}{2fC}$$

Hence

$$\Delta V = \frac{I}{fC} \left( \frac{2}{3} n^3 - \frac{n}{6} \right) \tag{2.10}$$

If $n \geq 4$ we find that the linear term can be neglected and, therefore, the voltage drop can be approximated to

$$\Delta V \approx \frac{I}{fC} \cdot \frac{2}{3} n^3 \tag{2.11}$$

The maximum output voltage is given by

$$V_{0 \ max} = 2nV_{\ max} - \frac{I}{fC} \cdot \frac{2}{3} n^3 \tag{2.12}$$

From (2.12) it is clear that for a given number of stages, a given frequency and capacitance of each stage, the output voltage decrease linearly with load current $I$. For a given load, however, $V_0 = (V_{0\ max} - V)$ may rise initially with the number of stages $n$, and reaches a maximum value but decays beyond on optimum number of stage. The optimum number of stages assuming a constant $V_{\ max}$, $I$, $f$ and $C$ can be obtained for maximum value of $V_0$ by differentiating equation (2.12) with respect to $n$ and equating it to zero.
\[ \frac{dV_{\max}}{dh} = 2V_{\max} - \frac{2}{3} \frac{I}{fC} n^2 = 0 \]

or

\[ n_{\text{opt}} = \sqrt{\frac{V_{\max} fC}{I}} \]  

(2.13)

Substituting \( n_{\text{opt}} \) in equation (2.12) we have

\[ (V_{0 \ max})_{\max} = \sqrt{\frac{V_{\max} fC}{I}} \left( 2V_{\max} - \frac{2I}{3fC} \frac{V_{\max} fC}{I} \right) \]

\[ = \sqrt{\frac{V_{\max} fC}{I}} \left( 2V_{\max} - \frac{2}{3} V_{\max} \right) \]

\[ = \sqrt{\frac{V_{\max} fC}{I}} \cdot \frac{4}{3} V_{\max} \]  

(2.14)

It is to be noted that in general it is more economical to use high frequency and smaller value of capacitance to reduce the ripples or the voltage drop rather than low frequency and high capacitance.

Cascaded generators of Cockroft-Walton type are used and manufactured worldwide these days. A typical circuit is shown in Fig. 2.5. In general a direct current up to 20 mA is required for high voltages between 1 MV and 2 MV. In case where a higher value of current is required, symmetrical cascaded rectifiers have been developed. These consist of mainly two rectifiers in cascade with a common smoothing column. The symmetrical cascaded rectifier has a smaller voltage drop and also a smaller voltage ripple than the simple cascade. The alternating current input to the individual circuits must be provided at the appropriate high potential; this can be done by means of isolating transformer. Fig. 2.6 shows a typical cascaded rectifier circuit. Each stage consists of one transformer which feeds two half wave rectifiers.
Fig. 2.5 A typical Cockroft circuit

Fig. 2.6 Cascaded rectifier circuit
As the storage capacitors of these half wave rectifiers are series connected even the h.v. winding of $T_1$ can not be grounded. This means that the main insulation between the primary and the secondary winding of $T_1$ has to be insulated for a d.c. voltage of magnitude $V_{max}$, the peak voltage of $T_1$. The same is required for $T_2$ also but this time the high voltage winding is at a voltage of $3V_{max}$. It would be difficult to provide the whole main insulation within this transformer, an isolating transformer $T$ supplies $T_2$. The cascading of every stage would thus require an additional isolating transformer which makes this circuit less economical for more than two stages.

**ELECTROSTATIC GENERATOR**

In electromagnetic generators, current carrying conductors are moved against the electromagnetic forces acting upon them. In contrast to the generator, electrostatic generators convert mechanical energy into electric energy directly. The electric charges are moved against the force of electric fields, thereby higher potential energy is gained at the cost of mechanical energy. The basic principle of operation is explained with the help of Fig. 2.7.

![Fig. 2.7](image)

An insulated belt is moving with uniform velocity $v$ in an electric field of strength $E$ $(x)$. Suppose the width of the belt is $b$ and the charge density $\sigma$ consider a length $dx$ of the belt, the charge $dq = \sigma \ b dx$. The force experienced by this charge (or the force experienced by the belt).
Assuming no losses, the power output is also equal to \( VI \).

Fig. 2.8 shows belt driven electrostatic generator developed by Van de Graaf in 1931. An insulating belt is run over pulleys. The belt, the width of which may vary from a few cms to metres is driven at a speed of about 15 to 30 m/sec, by means of a motor connected to the lower pulley. The belt near the lower pulley is charged electrostatically by an excitation arrangement. The lower charge spray unit consists of a number of needles connected to the controllable d.c. source (10 kV–100 kV) so that the discharge between the points and the belt is maintained. The charge is conveyed to the upper end where it is collected from the belt by discharging points connected to the inside of an insulated metal electrode through which the belt passes. The entire equipment is enclosed in an earthed metal tank filled with insulating gases of good dielectric strength viz. SF6 etc. So that the potential of the electrode could be raised to relatively higher voltage without corona discharges or for a certain voltage a smaller size of the equipment will result. Also, the shape of the h.t., electrode should be such that the surface gradient of electric field is made uniform to reduce again corona discharges, even though it is desirable to avoid corona entirely. An isolated sphere is the most favourable electrode shape and will maintain a uniform field \( E \) with a voltage of \( Er \) where \( r \) is the radius of the sphere.
As the h.t. electrode collects charges its potential rises. The potential at any instant is given as \( V = \frac{q}{C} \) where \( q \) is the charge collected at that instant. It appears as though if the charge were collected for a long time any amount of voltage could be generated. However, as the potential of electrode rises, the field set up by the electrode increases and that may ionise the surrounding medium and, therefore, this would be the limiting value of the voltage. In practice, equilibrium is established at a terminal voltage which is such that the charging current

\[
I = C \frac{dV}{dt}
\]

equals the discharge current which will include the load current and the leakage and corona loss currents. The moving belt system also distorts the electric field and, therefore, it is placed within properly shaped field grading rings. The grading is provided by resistors and additional corona discharge elements. The collector needle system is placed near the point where the belt enters the h.t. terminal. A second point system excited by a self-inducing arrangement enables the down going belt to be charged to the polarity opposite to that of the terminal and thus the rate of charging

![Fig. 2.8 Van de Graaf generator](image)
of the latter, for a given speed, is doubled. The self inducing arrangement requires insulating the upper pulley and maintaining it at a potential higher than that of the h.t. terminal by connecting the pulley to the collector needlesystem. The arrangement also consists of a row of points (shown as upper spray points in Fig. 2.8) connected to the inside of the h.t. terminal and directed towards the pulley above its points of entry into the terminal. As the pulley is at a higher potential (positive), the negative charges due to corona discharge at the upper spray points are collected by the belt. This neutralises any remaining positive charge on the belt and leaves an excess of negative charges on the down going belt to be neutralised by the lower spray points. Since these negative charges leave the h.t. terminal, the potential of the h.t. terminal is raised by the corresponding amount.

**GENERATION OF HIGH A.C. VOLTAGES**

Most of the present day transmission and distribution networks are operating on a.c. voltages and hence most of the testing equipments relate to high a.c. voltages. Even though most of the equipments on the system are 3-phase systems, a single phase transformer operating at power frequency is the most common from of HVAC testing equipment. Test transformers normally used for the purpose have low power rating but high voltage ratings. These transformers are mainly used for short time tests on high voltage equipments. The currents required for these tests on various equipments are given below:

| Insulators, C.B., bushings, Instrument transformers | = 0.1–0.5 A |
| Power transformers, h.v. capacitors. | = 0.5–1 A |
| Cables | = 1 A and above |

**Cascaded Transformers**

For voltages higher than 400 KV, it is desired to cascade two or more transformers depending upon the voltage requirements. With this, the weight of the whole unit is subdivided into single units and, therefore, transport and erection becomes easier. Also, with this, the transformer cost for a given voltage may be reduced, since cascaded units need not individually possess the expensive and heavy insulation required in single stage transformers for high voltages exceeding 345 kV. It is found that the cost of insulation for such voltages for a single unit becomes proportional to square of operating voltage.
Fig. 2.9 shows a basic scheme for cascading three transformers. The primary of the first stage transformer is connected to a low voltage supply. A voltage is available across the secondary of this transformer. The tertiary winding (excitation winding) of first stage has the same number of turns as the primary winding, and feeds the primary of the second stage transformer. The potential of the tertiary is fixed to the potential $V$ of the secondary winding as shown in Fig. 2.9. The secondary winding of the second stage transformer is connected in series with the secondary winding of the first stage transformer, so that a voltage of 2V is available between the ground and the terminal of secondary of the second stage transformer. Similarly, the stage-III transformer is connected in series with the second stage transformer. With this the output voltage between ground and the third stage transformer, secondary is 3V. it is to be noted that the individual stages except the upper most must have three-winding transformers. The upper most, however, will be a two winding transformer.

Fig. 2.9 shows metal tank construction of transformers and the secondary winding is not divided. Here the low voltage terminal of the secondary winding is connected to the tank. The tank of stage-I transformer is earthed. The tanks of stage-II and stage-III transformers have potentials of $V$ and 2V, respectively above earth and, therefore, these must be insulated from the earth with suitable solid insulation. Through h.t. bushings, the leads from the tertiary winding and the h.v. winding are brought out to be connected to the next stage transformer.
However, if the high voltage windings are of mid-point potential type, the tanks are held at 0.5 V, 1.5 V and 2.5 V, respectively. This connection results in a cheaper construction and the high voltage insulation now needs to be designed for V/2 from its tank potential.

The main disadvantage of cascading the transformers is that the lower stages of the primaries of the transformers are loaded more as compared with the upper stages. The loading of various windings is indicated by P in Fig. 2.9. For the three-stage transformer, the total output VA will be $3VI = 3P$ and, therefore, each of the secondary winding of the transformer would carry a current of $I = P/V$. The primary winding of stage-III transformer is loaded with P and so also the tertiary winding of second stage transformer. Therefore, the primary of the second stage transformer would be loaded with 2P. Extending the same logic, it is found that the first stage primary would be loaded with P. Therefore, while designing the primaries and tertiaries of these transformers, this factor must be taken into consideration.

![Equivalent Circuit of One Stage](image)

**Fig. 2.10** Equivalent circuit of one stage

The total short circuit impedance of a cascaded transformer from data for individual stages can be obtained. The equivalent circuit of an individual stage is shown in Fig. 2.10.
Also let \( N_p = N_t \) for all stages, the equivalent circuit for a 3-stage transformer would be given as in Fig. 2.11.

Fig. 2.11 Equivalent circuit of 3-stage transformer

\[
Z_p = jX_p, \quad Z_s = jX_s \quad \text{and} \quad Z_t = jX_t
\]

\[
V_1 = \frac{3N_s}{N_p} V_1
\]

Fig. 2.12 A simplified equivalent circuit

Fig. 2.11 can be further reduced to a very simplified circuit as shown in Fig. 2.12. The resulting short circuit reactance \( X_{res} \) is obtained from the condition that the power rating of the two circuits be the same. Here currents have been shown corresponding to high voltage side.