**Permeability**

Permeability is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids. The rock permeability, \( k \), is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by Henry Darcy in 1856. In fact, the equation that defines permeability in terms of measurable quantities is called Darcy’s Law.

Note: permeability is an intensive property of a porous medium (e.g. reservoir rock)

**Darcy's Law**

Or the following figure

The pressure at any point in the flow path, fig. 4.2, which has an elevation \( z \), relative to the datum plane, can be expressed in absolute units as

\[
p = \rho g (h - z)
\]

\[
hg = \left( \frac{p}{\rho} + gz \right)
\]
Then the velocity is written as

\[ u = \frac{K}{g} \frac{dh}{dl} \]

Let the potential of fluid defined as

\[ \Phi = \frac{p}{\rho} + gz \]

The constant \( \frac{K}{g} \) is only applicable for the flow of water, which was the liquid used exclusively in Darcy's experiments. Experiments performed with a variety of different liquids revealed that the law can be generalised as

\[ u = \frac{k\rho}{\mu} \frac{d\Phi}{dl} \]
In general the pressure \( p = \rho \Phi \) the Darcy law can be written as

\[
v = -\frac{k}{\mu} \frac{dp}{dL}
\]

where \( v \) = apparent fluid flowing velocity, cm/sec
\( k \) = proportionality constant, or permeability, Darcy’s
\( \mu \) = viscosity of the flowing fluid, cp
\( \frac{dp}{dL} \) = pressure drop per unit length, atm/cm

If \( q = uA \) then

\[
q = -\frac{kA}{\mu} \frac{dp}{dL}
\]

where \( q \) = flow rate through the porous medium, cm\(^3\)/sec
\( A \) = cross-sectional area across which flow occurs, cm\(^2\)

- Permeability is a derived dimension
- From Darcy’s equation, the dimension of permeability is length squared
  \[ k = \frac{q \mu L}{A \Delta p} ; \quad \left[ \frac{L^3}{T \cdot 1 \cdot 1} \right] = [L^2] \]
  - This is not the same as area, even though for example, it is m\(^2\) in SI units
- In Darcy and SI Units, this equation is coherent
  - Oilfield units are non-coherent, a unit conversion constant is required

- Permeability in d
- viscosity in cp
- length in cm
- volume in cm\(^3\)
- pressure in atm
- time in s

\[
v_s = -\frac{k}{\mu} \frac{\Delta p}{\Delta x}
\]
• Permeability has a derived dimension L^2 based on Darcy’s Equation:

\[ v_s = -\frac{k}{\mu} \frac{\Delta p}{\Delta x} \quad \left[ \frac{m}{s} \right] = \left[ \frac{m^2}{Pa \cdot s} \times \frac{Pa}{m} \right] \]

• The SI unit of permeability is 1 m^2
  – The oilfield unit is millidarcy
  – 1 md = 9.87 \times 10^{-16} m^2

• If you have 1 m long core with 9.87 \times 10^{-16} m^2 permeability and you apply 10^5 Pa pressure difference to the two ends, the Darcy velocity of water (\(\mu=0.001\) Pa-s) will be \(v_s = 9.87 \times 10^{-8}\) m/s

• If the core has a cross sectional area 0.1 m^2, the flow rate will be

\[ q = \frac{k A \Delta p}{\mu \Delta L} \]

\[ \frac{cm^3}{s} = \frac{-k \ cm^2}{0.001 \ Pa \cdot s} \frac{101325 \ Pa}{cm} \]

\[ k = 9.87 \times 10^{-9} \ cm^2 \]

\[ d = 9.87 \times 10^{-9} \ cm^2 \]

\[ d = 9.87 \times 10^{-13} \ m^2 \]

\[ md = 9.87 \times 10^{-16} \ cm^2 \]

\[ v_s = -7.324 \times 10^{-7} \times \frac{k}{\mu} \times \frac{\Delta p}{\Delta x} \quad \left[ \frac{ft}{s} \right] \left[ \frac{md}{cp} \right] \left[ \frac{psi}{ft} \right] \]

• If you have 3 ft long core with 1 md permeability and you apply 14.7 psi pressure difference to the two ends, the Darcy velocity of water (\(\mu = 1\) cP) will be \(v_s = 3.59 \times 10^{-7}\) ft/s

• If the core has a cross sectional area 1 ft^2, the flow rate will be \(3.59 \times 10^{-7}\) ft^3/s
\[ v_s (ft/s) = \frac{1}{30.5} \times 10^{-3} \frac{md}{cp} \frac{(1/14.7) psi}{30.5 \text{ ft}} \]

\[ v_s (ft/s) = -7.324 \times 10^{-8} \times \frac{k(md)}{\mu(cp)} \frac{\Delta p(psi)}{\Delta x(ft)} t \]

\[ q = 1.127 \times 10^{-3} \times A \times \frac{k}{\mu} \frac{\Delta p}{\Delta x} \]

- If you have 3 ft long core with 1 md permeability and you apply 14.7 psi pressure difference to the two ends, and the flowing fluid is water ($\mu = 1 \text{ cP}$) and the core has a cross sectional area 1 ft², the flow rate will be 0.00552 bbl/day.

Permeability classification

**Absolute Permeability (k):**

Permeability of a rock to a fluid when the rock is 100% saturated with that fluid. It is the permeability given by Dary’s Law:

\[ q = \frac{AK \frac{dP}{dr}}{\mu} \]
When several fluids are flowing through a porous medium, the flowrate of each fluid will be governed by Darcy’s law. Basically, such flowrate is dictated by the fluid’s viscosity and flow potential gradient, the portion of the total cross-sectional area of the medium that is available to the fluid’s flow, and the permeability of the medium. In equation form, Darcy’s law is written:

\[ q_i = -k \frac{A_i}{\mu_i} \frac{\partial \Phi_i}{\partial s} \]

where subscript “i” refers to fluid i. Since \( A_i \) is difficult and impractical to determine, the total cross-sectional area of the medium is preferred instead. This necessitates replacing k with \( k_i \), which is termed the effective permeability to fluid i. The above equation is, therefore, rewritten:

\[ q_i = -k_i \frac{A}{\mu_i} \frac{\partial \Phi_i}{\partial s} \]

The effective permeability to a fluid is, thus, defined as the ability of a porous medium to conduct that fluid when the fluid’s saturation in the porous medium is less than 100%.
Measurement of permeability

Permeability is measured by passing a fluid of known viscosity through a core sample of measured dimensions and then measuring flow rate and pressure drop. Various techniques are used for permeability measurements of cores, depending on sample dimensions and shape, degree of consolidation, type of fluid used, ranges of confining and fluid pressure applied, and range of permeability of the core. Two types of instruments are usually used in the laboratory:

(a) Variable head permeameter, IFP type.
(b) Constant head permeameter, Core Laboratories type.

Permeability tests are performed on samples which have been cleaned and dried and a gas (usually air) is used for flowing fluid in the test. This is because:

1. steady state is obtained rapidly,
2. dry air will not alter the minerals in the rock, and
3. 100% fluid saturation is easily obtained.

Measured values using constant head equipment range from a low of 0.1 mD to 20 D. Data accuracy declines at high and low permeability values.

This equipment is designed for plug or whole core permeability measurements. This experiment may be used for single or multiphase, compressible fluid or liquid measurements and can also be used under reservoir pressure and temperature.

Hassler core holder may be used with this instrument. The Hassler system is an improvement of the rubber plug system whose tightness is limited at certain pressures. The core is placed in a flexible rubber tube. The Hassler cell has these advantages:

- Excellent tightness.
- Can be used for samples of different sizes.
- Much higher pressure or $P$ can be used.
- Can be used for measuring relative permeability.
Constant Head Permeameter

Flow potential

- The generalized form of Darcy’s Law includes pressure and gravity terms to account for horizontal or non-horizontal flow

\[ v_s = \frac{q_s}{A} = -\frac{k}{\mu} \left[ \frac{dp}{ds} - \rho g \frac{dz}{ds} \right] \]

- The gravity term has dimension of pressure / length

- Flow potential includes both pressure and gravity terms, simplifying Darcy’s Law

\[ v_s = \frac{q}{A} = -\frac{k}{\mu} \left[ \frac{d\Phi}{ds} \right] \]

- \( \Phi = p - \rho g Z/c \); \( Z \uparrow \); \( Z \) is elevation measured from a datum
- \( \Phi \) has dimension of pressure
Linear flow, incompressible liquid

• **1-D Linear Flow System Assumptions**
  - steady state flow
  - incompressible fluid, \( q(0 \leq s \leq L) = \text{constant} \)
  - \( d\Phi \) includes effect of \( dZ/ds \) (change in elevation)
  - \( A(0 \leq s \leq L) = \text{constant} \)
  - Darcy flow (Darcy’s Law is valid)
  - \( k = \text{constant} \) (non-reactive fluid)
  - single phase (\( S=1 \))
  - isothermal (constant \( \mu \))

  ![Diagram of linear flow system](image)

• Darcy’s Law:

\[
\begin{align*}
\nu_s &= \frac{q}{A} = -\frac{k}{\mu} \left[ \frac{d\Phi}{ds} \right] \\
q \ ds &= -\frac{kA}{\mu} d\Phi \\
q \int_0^L ds &= -\frac{kA}{\mu} \int_{\Phi_1}^{\Phi_2} d\Phi \\
q &= \frac{k A}{\mu L} (\Phi_1 - \Phi_2)
\end{align*}
\]

- \( q_{1 \rightarrow 2} > 0 \), if \( \Phi_1 > \Phi_2 \)
- Use of flow potential, \( \Phi \), valid for horizontal, vertical or inclined flow

---

**If there is change in elevation:**

\[
\frac{dZ}{ds} = 0.0
\]

Then

\[
q = \frac{Ak}{\mu L} \frac{dP}{dL} = \frac{Ak(P_1 - P_2)}{\mu L}
\]
Radial flow, incompressible liquid

- **1-D Radial Flow System Assumptions**
  - steady state flow
  - incompressible fluid, \( q(r_w \leq s \leq r_e) = \text{constant} \)
  - horizontal flow \( (dZ/ds = 0 \therefore \Phi = p) \)
  - \( A(r_w \leq s \leq r_e) = 2\pi rh \) where, \( h = \text{constant} \)
  - Darcy flow (Darcy’s Law is valid)
  - \( k = \text{constant} \) (non-reactive fluid)
  - single phase \( (S=1) \)
  - isothermal (constant \( \mu \))
  - \( ds = -dr \)

Radial Flow, Incompressible Liquid

- **Darcy’s Law:**
  \[
  v_s = \frac{q}{A} = -\frac{k}{\mu} \left[ \frac{d\Phi}{ds} \right]
  \]
  \[
  \frac{q}{2\pi rh} dr = \frac{k}{\mu} dp
  \]
  \[
  q \int_{r_e}^{r_w} \frac{1}{r} dr = \frac{2\pi kh}{\mu} \int_{p_w}^{p_e} dp
  \]
  \[
  q = \frac{2\pi kh}{\mu \ln(r_e/r_w)} (p_e - p_w)
  \]
  \( q_{e \rightarrow w} > 0, \) if \( p_e > p_w \)
Example 1

A brine is used to measure the absolute permeability of a core plug. The rock sample is 4 cm long and 3 cm² in cross section. The brine has a viscosity of 1.0 cp and is flowing a constant rate of 0.5 cm³/sec under a 2.0 atm pressure differential. Calculate the absolute permeability.

Solution

Applying Darcy’s equation,

\[ 0.5 = \frac{(k)(3)(2)}{(1)(4)} \]

\[ k = 0.333 \text{ Darcys} \]

Example 2

Rework the above example assuming that an oil of 2.0 cp is used to measure the permeability. Under the same differential pressure, the flow rate is 0.25 cm³/sec.

Solution

Applying Darcy’s equation yields:

\[ 0.25 = \frac{(k)(3)(2)}{(2)(4)} \]

\[ k = 0.333 \text{ Darcys} \]

Example 3

What is the flow rate of a horizontal rectangular system when the conditions are as follows:
permeability = \( k = 1 \) darcy
area = \( A = 6 \text{ ft}^2 \)
viscosity = \( m = 1.0 \text{ cp} \)
length = L = 6 ft
inlet pressure = P1 = 5.0 atm
outlet pressure = P2 = 2.0 atm

Solution:
We must insure all the variables are in the correct units.

\[ k = 1 \text{ darcy} \]
\[ A = 6 \text{ ft}^2 (144 \text{ in}^2/1 \text{ ft}^2) (6.45 \text{ cm}^2/1 \text{ in}^2) = 5572.8 \text{ cm}^2 \]
\[ L = 6 \text{ ft} (12 \text{ in}/1 \text{ ft}) (2.54 \text{ cm}/1 \text{ in}) = 182.88 \text{ cm} \]
\[ P1 = 5.0 \text{ atm} \]
\[ P2 = 2.0 \text{ atm} \]

\[ q = \frac{kA}{L \mu} (P2 - P1) \]

\[ q = \left( \frac{1}{1} \right) \left( \frac{5572.8}{1} \right) \left( \frac{5.0 - 2.0}{1} \right) \]

\[ q = 91.42 \text{ cm}^3/\text{sec} \]

\textbf{Example 4}
What is the flow rate of a horizontal rectangular system when the conditions are as follows:
permeability = $k = 1$ Darcy
area = $A = 6 \text{ ft}^2$
viscosity = $\mu = 1.0 \text{ cp}$
length = $L = 6 \text{ ft}$
inlet pressure = $P1 = 5.0 \text{ atm}$
outlet pressure = $P2 = 2.0 \text{ atm}$

Solutions:
We must insure that all the variables are in the correct units.

\[ k = 1 \text{ darcy} = 1,000 \text{ md} \]
\[ A = 6 \text{ ft}^2 \]
\[ L = 6 \text{ ft} \]
\[ P1 = (5.0 \text{ atm}) (14.7 \text{ psi/atm}) = 73.5 \text{ psi} \]
\[ P2 = (2.0 \text{ atm}) (14.7 \text{ psi/atm}) = 29.4 \text{ psi} \]

\[ q = 1.1271 \times 10^{-3} \frac{kA}{\mu L} (P1 - P2) \]

\[ q = 1.1271 \times 10^{-3} \left( \frac{1,000}{1} \right) \left( \frac{6}{1} \right) \left( 73.5 - 29.4 \right) \]

\[ q = 49.7 \text{ bbl/ day} \]
**Example 5**

Find the relation of permeability for inclined pipe as shown below:

Where \( Z: \text{negative}, \ z = s \sin \theta \)

From the relation of pressure \( \Phi = P - \rho g z \)

\[
\frac{d\Phi}{ds} = \frac{dP}{ds} - \rho g \sin \theta
\]

Substituting in the Darcy's law

\[
q = -k \frac{A}{\mu} \left[ \frac{dP}{ds} - \rho g \sin \theta \right]
\]

Separating of the variables \( p \) and \( s \) and then integration

\[
\int_0^L \left[ \frac{q \mu}{kA} - \rho g \sin \theta \right] ds = - \int_{P_1}^{P_2} dP
\]

obtain

\[
\left[ \frac{q \mu}{kA} - \rho g \sin \theta \right] L = P_1 - P_2
\]

Rearrangement the above equation gives

\[
q = \frac{k A}{\mu} \frac{(P_1 - P_2) + \rho g L \sin \theta}{L}
\]

Or

\[
q = 1.127 \frac{k A}{\mu} \frac{(P_1 - P_2) + \frac{\rho}{144} L \sin \theta}{L}
\]
Example 6
A sandstone aquifer 8 miles long, 2 miles wide and 70 feet thick with a dip angle of 6 degrees and permeability of 80 md is conducting water from the ocean floor to a reservoir at the other end as shown in Fig. 6.7. The ocean floor is 600 feet deep, and sea water has a density of 68 lb/ft$^3$ and viscosity of 1.4 cp. If the reservoir pressure is 1600 psia, compute the steady-state rate at which the aquifer is charging water into the reservoir.

$$A = 2 \times 5280 \times 70 = 739200 \text{ ft}^2$$

The inlet pressure is the hydrostatic pressure at the ocean floor

$$P_1 = \frac{68 \times 600}{144} + 14.7 = 298 \text{ psia}$$

Since the aquifer’s outlet pressure is equal to the reservoir pressure, water flow rate through the aquifer is computed by the above equation as

$$q = 1.127 \frac{0.08 \times 739200}{1.4} \frac{(298 - 1600) + \frac{68}{144} \times 8 \times 5280 \sin 6}{8 \times 5280}$$

$$= 882.4 \text{ bbl/d}$$

Fig. 6.7: Aquifer of Example 6.5
Example 7
A horizontal reservoir is nearly circular in shape with an area of 730 acres, thickness of 120 feet and permeability of 250 md. One well penetrates the reservoir at its approximate center with a diameter of 9". If the pressure is 3200 psig at the periphery of the reservoir and 1500 psig at the wellbore; and if the reservoir oil is 2.5 cp in viscosity, compute the well’s daily production rate assuming all pressures remain constant with time. Since the reservoir is nearly radial, we can compute an equivalent radius by the following approximation:

\[
    r_e = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{730 \times 43560}{\pi}} = 3181.5 \text{ ft}
\]

\[
    r_w = \frac{9}{12 \times 2} = 0.375 \text{ ft}
\]

\[
    q = 7.082 \frac{0.25 \times 120}{2.5} \frac{3200 - 1500}{\ln \frac{3181.5}{0.375}} = 15971 \text{ bbl/d}
\]

Homework
1. A cylindrical core having a radius \(2.54 \times 10^{-2} \text{ [m]}\) and length of 0.3 [m], was flooded with brine at a steady rate of \(1.10^{-6} \text{ [m}^3\text{s}^{-1}]\), the differential pressure across the core was 10 [bar]. Calculate the absolute permeability of the core. Assume brine viscosity 0.001 [Pa.s].

2. Calculate the permeability of a core plug from the following test:
   - Flow rate = \(2.10^{-6} \text{ [m}^3\text{s}^{-1}]\)
   - Inlet pressure = 5 [bar]
   - Outlet pressure = 1 [bar]
   - Length of core = 0.1 [m]
   - Area = \(1.10^{-4} \text{ [m}^2]\)
   - Viscosity = 0.002 [Pa.s]

3. If \(10^{-6} \text{ m}\) is called a micrometer (μm) and \(10^{-12} \text{ m}^2\) is called a micrometer squared (μm²), convert 230 md to μm².
4. Water ($\mu = 1 \text{ cP}$) is flowing through a core sample ($L = 10 \text{ cm}$, $D = 2.5 \text{ cm}$) of 170 md permeability. Compute the flowrate if inlet and outlet pressures are 5 and 2 atm, respectively. Give your answer in cm$^3$/s and bbl/d.

5. Compute the absolute pressure at the bottom of the Arabian Gulf (depth = 820 ft) if seawater has an average density of 64 lb/ft$^3$ and atmospheric pressure is 14.7 psia.

6- A linear reservoir is 7 km long, 2 km wide and 20 m thick. It has a porosity of 22%, permeability of 350 md and it is inclined at 4° with the horizontal plain. If water ($\rho = 1 \text{ g/cm}^3$, $\mu = 1 \text{ cP}$) enters the reservoir at a pressure of 150 atm, flows downwards through it, and exits the reservoir at 75 atm pressure. Compute the steady-state flowrate of water in m$^3$/d and in bbl/d.

7- A well with a diameter of 9 3/8 inches is drilled through an 80-ft thick reservoir with $k = 220 \text{ md}$. If the bottom-hole pressure of the well is 2400 psig, and if the pressure 3000 feet away from the well (in all directions) is 5000 psig, what will the oil production rate be? Assume $\mu_o = 2.5 \text{ cP}$. 