Chapter 8

Three Phase Induction Motor

8.1 Introduction

The popularity of 3 phase induction motors on board ships is because of their simple, robust construction, and high reliability factor in the sea environment. A 3 phase induction motor can be used for different applications with various speed and load requirements. Electric motors can be found in almost every production process today. Getting the most out of your application is becoming more and more important in order to ensure cost-effective operations. The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a “transformer type” a.c. machine in which electrical energy is converted into mechanical energy.

8.1.1 Advantages

(i) It has simple and rugged construction.

(ii) It is relatively cheap.

(iii) It requires little maintenance.
(iv) It has high efficiency and reasonably good power factor.

(v) It has self-starting torque.

### 8.1.2 Disadvantages

(i) It is essentially a constant speed motor and its speed cannot be changed easily.

(ii) Its starting torque is inferior to d.c. shunt motor.

### 8.2 Construction

The three phase induction motor is the most widely used electrical motor. Almost 80% of the mechanical power used by industries is provided by three phase induction motors because of its simple and rugged construction, low cost, good operating characteristics, absence of commutator and good speed regulation. In three phase induction motor the power is transferred from stator to rotor winding through induction. The Induction motor is also called asynchronous motor as it runs at a speed other than the synchronous speed. Like any other electrical motor induction motor also have two main parts namely rotor and stator. A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor. The main body of the Induction Motor comprises of two major parts as shows in Figure 1:

i. Shaft for transmitting the torque to the load. This shaft is made up of steel.

ii. Bearings for supporting the rotating shaft.

iii. One of the problems with electrical motor is the production of heat during its rotation. In order to overcome this problem we need fan for cooling.

iv. For receiving external electrical connection Terminal box is needed.

v. There is a small distance between rotor and stator which usually varies from 0.4 mm to 4 mm. Such a distance is called air gap.
8.2.1. Stator

Stator: As its name indicates stator is a stationary part of induction motor. A stator winding is placed in the stator of induction motor and the three phase supply is given to it. Stator is made up of number of stampings in which different slots are cut to receive 3 phase winding circuit which is connected to 3 phase AC supply. The three phase windings are arranged in such a manner in the slots that they produce a rotating magnetic field after AC supply is given to them. The windings are wound for a definite number of poles depending upon the speed requirement, as speed is inversely proportional to the number of poles, given by the formula:

$$N_s = \frac{120f}{p}$$

Where $N_s$ = synchronous speed

$f$ = Frequency

$p$ = no. of poles
It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations [See Fig. (8.2)]. The insulated connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

8.2.1.1 Stator of Three Phase Induction Motor

The stator of the three phase induction motor consists of three main parts :

i. **Stator Frame**

It is the outer most part of the three phase induction motor. Its main function is to support the stator core and the field winding. It acts as a covering and it provide protection and mechanical strength to all the inner parts of the induction motor. The frame is either made up of die cast or fabricated steel. The frame of three phase induction motor should be very strong and rigid as the air gap length of motorist very small, otherwise rotor will not remain concentric with stator, which will give rise to unbalanced magnetic pull.
ii. **Stator Core**

The main function of the stator core is to carry the alternating flux. In order to reduce the eddy current loss, the stator core is laminated. These laminated types of structure are made up of stamping which is about 0.4 to 0.5 mm thick. All the stamping are stamped together to form stator core, which is then housed in stator frame. The stamping is generally made up of silicon steel, which helps to reduce the hysteresis loss occurring in motor.

iii. **Stator Winding or Field Winding**

The slots on the periphery of stator core of the motor carries three phase windings. This three phase winding is supplied by three phase ac supply. The three phases of the winding are connected either in star or delta depending upon which type of starting method is used. The squirrel cage motor is mostly started by star – delta stator and hence the stator of squirrel cage motor is delta connected. The slip ring three phase induction motor are started by inserting resistances so, the stator winding of slip ring induction can be connected either in star or delta. The winding wound on the stator of three phase induction motor is also called field winding and when this winding is excited by three phase ac supply it produces a rotating magnetic.

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8.2.2. **Rotor**

The rotor is a rotating part of induction motor. The rotor is connected to the mechanical load through the shaft. Rotor consists of cylindrical laminated core with parallel slots that carry conductor bars. Conductors are heavy copper or aluminium bars which fits in each slots. These conductors are brazed to the short circuiting end rings. The slots are not exactly made parallel to the axis of the shaft but are slotted a little skewed for the following reason, They reduces magnetic hum or noise and They avoid stalling of motor. The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types: Squirrel cage type and Wound type
8.2.2.1 Squirrel cage rotor.

Squirrel cage three phase induction motor: The rotor of the squirrel cage three phase induction motor is cylindrical in shape and have slots on its periphery. The slots are not made parallel to each other but are bit skewed (skewing is not shown in the figure of squirrel cage rotor beside) as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of motor more smooth and quieter. The squirrel cage rotor consists of aluminum, brass or copper bars. These aluminum, brass or copper bars are called rotor conductors and are placed in the slots on the periphery of the rotor. The rotor conductors are permanently shorted by the copper or aluminum rings called the end rings. In order to provide mechanical strength these rotor conductor are braced to the end ring and hence form a complete closed circuit resembling like a cage and hence got its name as “squirrel cage induction motor”. The squirrel cage rotor winding is made symmetrical. As the bars are permanently shorted by end rings, the rotor resistance is very small and it is not possible to add external resistance as the bars are permanently shorted. The absence of slip ring and brushes make the construction of Squirrel cage three phase induction motor very simple and robust and hence widely used three phase induction motor. These motors have the advantage of adapting any number of pole pairs. The below diagram shows squirrel cage induction rotor having aluminum bars short circuit by aluminum end rings. It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. (8.3)]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator. Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not
possible to add any external resistance to the rotor circuit to have a large starting torque.

![Fig. 8.3 squirrel cage rotor.](image)

**Advantages of squirrel cage induction rotor**

i. Its construction is very simple and rugged.

ii. As there are no brushes and slip ring, these motors requires less maintenance.

**Applications:**

Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc.

**8.2.2.2 Wound rotor.**

Slip ring or wound three phase induction motor: In this type of three phase induction motor the rotor is wound for the same number of poles as that of stator but it has less number of slots and has less turns per phase of a heavier conductor. The rotor also carries star or delta winding similar to that of stator winding. The rotor consists of numbers of slots and rotor winding are placed inside these slots. The three end terminals are connected together to form star connection. As its name indicates three phase slip ring induction motor consists of slip rings connected on same shaft as that of rotor. The three ends of three phase windings are permanently connected to these slip rings. The external resistance can be easily connected through the brushes and slip rings and hence used for speed control and improving the starting torque of three
phase induction motor. The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances. At starting, the resistance are connected in rotor circuit and is gradually cut out as the rotor pick up its speed. When the motor is running the slip ring are shorted by connecting a metal collar, which connect all slip ring together and the brushes are also removed. This reduces wear and tear of the brushes. Due to presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor. It consists of a laminated cylindrical core and carries a 3- phase winding, similar to the one on the stator [See Fig. (8.4)]. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig. (8.5). At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

Fig 8.4  Lamination of stator and rotor.
Advantages of slip ring induction motor -

A. It has high starting torque and low starting current.
B. Possibility of adding additional resistance to control speed.

Application:

Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc.

**Difference between Slip Ring and Squirrel Cage Induction Motor**

<table>
<thead>
<tr>
<th>SLIP RING OR PHASE WOUND</th>
<th>SQUIRREL CAGE</th>
</tr>
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<tbody>
<tr>
<td>Construction is complicated due to presence of slip ring and brushes</td>
<td>Construction is very simple</td>
</tr>
<tr>
<td>The rotor consists of rotor bars which are permanently shorted with the rotor winding is similar to the stator</td>
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winding the help of end rings

We can easily add rotor resistance by using slip ring and brushes. Since the rotor bars are permanently shorted, it is not possible to add external resistance.

Due to presence of external resistance, high starting torque can be obtained. Staring torque is low and cannot be improved.

<table>
<thead>
<tr>
<th>Slip ring and brushes are present</th>
<th>Slip ring and brushes are absent</th>
</tr>
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<tbody>
<tr>
<td>Frequent maintenance is required due to presence of brushes</td>
<td>Less maintenance is required</td>
</tr>
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</table>

The construction is complicated and the presence of brushes and slip ring makes the motor more costly. The construction is simple and robust and it is cheap as compared to slip ring induction motor.

This motor is rarely used only 10% industry uses slip ring induction motor. Due to its simple construction and low cost. The squirrel cage induction motor is widely used.

<table>
<thead>
<tr>
<th>Rotor copper losses are high and hence less efficiency</th>
<th>Less rotor copper losses and hence high efficiency</th>
</tr>
</thead>
</table>

Speed control by rotor resistance method is possible. Speed control by rotor resistance method is not possible.

Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc. Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc.
8.3 operation principle

Unlike toys and flashlights, most homes, offices, factories, and other buildings aren't powered by little batteries: they're not supplied with DC current, but with alternating current (AC), which reverses its direction about 50 times per second (with a frequency of 50 Hz). If you want to run a motor from your household AC electricity supply, instead of from a DC battery, you need a different design of motor.

In an AC motor, there's a ring of electromagnets arranged around the outside (making up the stator), which are designed to produce a rotating magnetic field. Inside the stator, there's a solid metal axle, a loop of wire, a coil, a squirrel cage made of metal bars and interconnections (like the rotating cages people sometimes get to amuse pet mice), or some other freely rotating metal part that can conduct electricity. Unlike in a DC motor, where you send power to the inner rotor, in an AC motor you send power to the outer coils that make up the stator. The coils are energized in pairs, in sequence, producing a magnetic field that rotates around the outside of the motor. The rotor, suspended inside the magnetic field, is an electrical conductor. The magnetic field is constantly changing (because it's rotating) so, according to the laws of electromagnetism (Faraday's law, to be precise), the magnetic field produces (or induces, to use Faraday's own term) an electric current inside the rotor. If the conductor is a ring or a wire, the current flows around it in a loop. If the conductor is simply a solid piece of metal, eddy currents swirl around it instead. Either way, the induced current produces its own magnetic field and, according to another law of electromagnetism (Lenz's law) tries to stop whatever it is that causes it—the rotating magnetic field—by rotating as well. (You can think of the rotor frantically trying to
"catch up" with the rotating magnetic field in an effort to eliminate the difference in motion between them.) Electromagnetic induction is the key to why a motor like this spins—and that's why it's called an induction motor. An electrical converts electrical energy into mechanical energy which is then supplied to different types of loads. A.C. motors operates on A.C. supply, and they are classified into synchronous, single phase and three phase induction, and special purpose motors. Out of all types, three phase induction motors are most widely used for industrial applications mainly because they do not require a starting device. three phase induction motor derives its name from the fact that the rotor current is induced by the magnetic field, instead of electrical connection. The operation principle of a three phase induction motors is based on the production of rotating magnetic field.

8.3.1 THREE-PHASE ROTATING FIELDS

The three-phase induction motor also operates on the principle of a rotating magnetic field. The following discussion shows how the stator windings can be connected to a three-phase ac input and have a resultant magnetic field that rotates.

Figure 8.6, views A-C show the individual windings for each phase. Figure 8.6, view D, shows how the three phases are tied together in a Y-connected stator. The dot in each diagram indicates the common point of the Y-connection. You can see that the individual phase windings are equally spaced around the stator. This places the windings 120° apart.
The three-phase input voltage to the stator of figure 8.6 is shown in the graph of figure 8.7. Use the left-hand rule for determining the electromagnetic polarity of the poles at any given instant. In applying the rule to the coils in figure 8.6, consider that current flows toward the terminal numbers for positive voltages, and away from the terminal numbers for negative voltages.
The results of this analysis are shown for voltage points 1 through 7 in figure 8.7. At point 1, the magnetic field in coils 1-1A is maximum with polarities as shown. At the same time, negative voltages are being felt in the 2-2A and 3-3A windings. These create weaker magnetic fields, which tend to aid the 1-1A field. At point 2, maximum negative voltage is being felt in the 3-3A windings. This creates a strong magnetic field which, in turn, is aided by the weaker fields in 1-1A and 2-2A. As each point on the voltage graph is analyzed, it can be seen that the resultant magnetic field is rotating in a clockwise direction. When the three-phase voltage completes one full cycle (point 7), the magnetic field has rotated through 360°.
8.3.2 Generation of rotating magnetic field (RMF)

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 φm where φm is the maximum flux due to any phase. Consider a three phase winding displaced in a space by 120° supplied by three phase A.C supply. The three phase current are also displaced from each other by 120°. The flux each phase current is also sinusoidal in nature and all three flux are separated from each other by 120°. If the phase sequence of winding is 1-2-3 (if the sequence between any two phases the direction of RMF will be anti clockwise), then the mathematical equation for the instantaneous values of the fluxes Φ1, Φ2, Φ3 can be given as:

\[ Φ1 = Φm \sin (wt) = Φm \sin ω \]

\[ Φ2 = Φm \sin (wt -120°) = Φm \sin (ω - 120°) \]

\[ Φ3 = Φm \sin (wt -240°) = Φm \sin (ω - 240°) \]

As windings are indicate and supply is balanced the amplitude of each flux is same i.e. Φm. The wave from three fluxes are shows in figure 8.8. While the assume positive direction of these fluxes in space are shown in Figure 8.9. Assume positive direction mean whenever the instantaneous value of the flux is positive vector diagram is must be represented along its assumed positive direction, and if flux has negative instantaneous value then must be represented in opposite direction to the assumed positive direction, in vector diagram.
Let \( \Phi_1, \Phi_2 \) and \( \Phi_3 \) be the instantaneous values of the fluxes. The resultant flux \( \Phi_T \), at any instant is given by phase combination of \( \Phi_1, \Phi_2 \) and \( \Phi_3 \) at the instant. Let us find out \( \Phi_T \) at four different instants 1, 2, 3 and 4 as shown in figure 8.9 i.e. respectively at \( \phi = \omega t = 0^\circ, 60^\circ, 120^\circ \) and \( 180^\circ \).

Case 1: when \( \phi = 0^\circ \)

\[ \Phi_1 = \Phi_m \sin (\omega t) = \Phi_m \sin 0^\circ = 0 \]

\[ \Phi_2 = \Phi_m \sin (\omega t - 120^\circ) = \Phi_m \sin (0^\circ - 120^\circ) = -0.866 \Phi_m \]
\[ \Phi_3 = \Phi_m \sin (w t - 240^\circ) = \Phi_m \sin (0^\circ - 240^\circ) = 0.866 \Phi_m \]

\[ \Phi_T = \Phi_1 + \Phi_2 + \Phi_3 \]

Hence vector diagram looks like as shown in Figure 8.10.

BD is perpendicular drawn from B on \( \Phi_T \)

Since \( OD = DA = \Phi_T / 2 \)

Since \( \Delta OBD \), the angle of \( BOD = 30^\circ \)

So \( \cos 30^\circ = OD/OB = (\Phi_T/2) / 0.866 \Phi_m \)

\[ \Phi_T = 2 \times 0.866 \Phi_m \times \cos 30^\circ = 1.5 \Phi_m \]

So the magnitude of resultant flux is 1.5 \( \Phi_m \) time the maximum value if flux.

Case 2: \( \omega = 60^\circ \)

\[ \Phi_1 = \Phi_m \sin (w t) = \Phi_m \sin 60^\circ = 0.866 \Phi_m \]

\[ \Phi_2 = \Phi_m \sin (w t - 120^\circ) = \Phi_m \sin (60^\circ - 120^\circ) = -0.866 \Phi_m \]

\[ \Phi_3 = \Phi_m \sin (w t - 240^\circ) = \Phi_m \sin (60^\circ - 240^\circ) = 0 \]
\( \Phi_T = \Phi_1 + \Phi_2 + \Phi_3 \)

Hence vector diagram looks like as shown in Figure 8.11.

BD is perpendicular drawn from B on \( \Phi_T \)

Since \( OD = DA = \Phi_T/2 \)

Since \( \Delta OBD \), the angle of \( BOD = 30^\circ \)

So \( \cos 30^\circ = OD/OB = (\Phi_T/2)/0.866 \Phi_m \)

\( \Phi_T = 2 \times 0.866 \Phi_m \times \cos 30^\circ = 1.5 \Phi_m \)

So the magnitude of resultant flux is 1.5 \( \Phi_m \) time the maximum value of flux.

---

Case 3: \( \omega = 120^\circ \)

\( \Phi_1 = \Phi_m \sin (\omega t) = \Phi_m \sin 120^\circ = 0.866 \Phi_m \)

\( \Phi_2 = \Phi_m \sin (\omega t - 120^\circ) = \Phi_m \sin (120^\circ - 120^\circ) = 0 \)

\( \Phi_3 = \Phi_m \sin (\omega t - 240^\circ) = \Phi_m \sin (120^\circ - 240^\circ) = -0.866 \Phi_m \)

\( \Phi_T = \Phi_1 + \Phi_2 + \Phi_3 \)

Hence vector diagram looks like as shown in Figure 8.12.

BD is perpendicular drawn from B on \( \Phi_T \)

Since \( OD = DA = \Phi_T/2 \)
Since \( \triangle OBD \), the angle of \( \angle BOD = 30^\circ \)

So \( \cos 30^\circ = \frac{OD}{OB} = \frac{\Phi_T/2}{0.866 \Phi_m} \)

\[ \Phi_T = 2 \times 0.866 \Phi_m \times \cos 30^\circ = 1.5 \Phi_m \]

So the magnitude of resultant flux is 1.5 \( \Phi_m \) time the maximum value if flux.

![Figure 8.11 vector diagram for angle 120°.](image)

Case 4 : \( \omega = 180^\circ \)

\( \Phi_1 = \Phi_m \sin (\omega t) = \Phi_m \sin 180^\circ = 0 \)

\( \Phi_2 = \Phi_m \sin (\omega t - 120^\circ) = \Phi_m \sin (180^\circ - 120^\circ) = 0.866 \Phi_m \)

\( \Phi_3 = \Phi_m \sin (\omega t - 240^\circ) = \Phi_m \sin (180^\circ - 240^\circ) = -0.866 \Phi_m \)

\( \Phi_T = \Phi_1 + \Phi_2 + \Phi_3 \)

Hence vector diagram looks like as shown in Figure 8.12.

BD is perpendicular drawn from B on \( \Phi_T \)

Since \( OD = DA = \Phi_T/2 \)

Since \( \triangle OBD \), the angle of \( \angle BOD = 30^\circ \)
So \( \cos 30^\circ = \frac{OD}{OB} = \frac{(\Phi_T/2)}{0.866 \Phi_m} \)

\[ \Phi_T = 2 \times 0.866 \Phi_m \times \cos 30^\circ = 1.5 \Phi_m \]

So the magnitude of resultant flux is 1.5 \( \Phi_m \) time the maximum value if flux.

![Vector diagram for angle 180°.](image)

**8.4 Speed of RMF**

The speed at which the rotating magnetic field revolves is called the synchronous speed (\( N_s \)). Referring to Fig. 8.9 the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, for \( P \) poles, the rotating field makes one revolution in \( P/2 \) cycles of current (see Fig. 8.7).

\[ \therefore \text{Cycles of current} = \frac{2}{P} \times \text{revolutions of field} \]

or \( \text{Cycles of current per second} = \frac{2}{P} \times \text{revolutions of field per second} \)

Since revolutions per second is equal to the revolutions per minute (\( N_s \)) divided by 60 and the number of cycles per second is the frequency (\( f \)).
The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

\[ f = \frac{P}{2} \times \frac{N_s}{60} = \frac{P N_s}{120} \]

\[ N_s = \frac{120 f}{P} \]

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

\[ f = \frac{P}{2} \times \frac{N_s}{60} = \frac{P N_s}{120} \]

\[ N_s = \frac{120 f}{P} \]

8.5 slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the stator field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

\[ \% \text{ age slip } S = \frac{N_s - N}{N_s} \times 100 \]

(i) The quantity Ns – N is sometimes called slip speed.

(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.

(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.
8.6 Rotor Frequency at operation condition

The frequency of a voltage or current induced due to the relative speed between a winding and a magnetic field is given by the general formula;

\[ f = \frac{P N}{120} \]

where \( N \) = Relative speed between magnetic field and the winding
\( P \) = Number of poles

For a rotor speed \( N \), the relative speed between the rotating flux and the rotor is \( N_s - N \). Consequently, the rotor current frequency \( f_2 \) is given by;

\[ f_2 = \frac{(N_s - N)P}{120} \]

\[ f_2 = \frac{S N_s P}{120} \]

\[ f_2 = S f_1 \]

Where \( f_2 \) = rotor current frequency, \( S \) = slip and \( f_1 \) = supply frequency (stator frequency). The relative speed between the rotating field and stator winding is \( N_s - 0 = N_s \). Therefore, the frequency of induced current or voltage in the stator winding is same as the supply frequency \( f_1 = N_s P/120 \).

8.7 Rotor current and power factor

Fig. (8.13) shows the circuit of a 3-phase induction motor at any slip \( s \). The rotor is assumed to be of wound type and star connected. Note that rotor e.m.f./phase and rotor reactance/phase are \( s E_2 \) and \( sX_2 \) respectively. The rotor resistance/phase is \( R_2 \) and is independent of frequency and, therefore, does not depend upon slip. Likewise, stator winding values \( R_1 \) and \( X_1 \) do not depend upon slip. Since the motor represents a balanced 3-phase load, we need consider one phase only; the conditions in the other two phases being similar.
At standstill. Fig. (8.14) shows one phase of the rotor circuit at standstill.

**Figure 8.14**

Rotor current \[ I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \]

**power factor** \[ \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \]

At running condition

Rotor current \[ I_2 = \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \]

**power factor** \[ \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \]
8.8 Rotor torque

The torque = force * radius

\[ T = F \times r \]

For one revolution (2\(\pi\))

\[ T = F \times r \times 2\pi \]

The power (P) = T * N

\[ P = F \times r \times 2\pi \times N \]

\[ P = T \times w \]

\[ T = P/W \]

\[ T = 1/w \times P \]

Where \( P = E \times I \times \cos\omega \)

\[ T = 1/w \times E \times I \times \cos\omega \]

For three phase

\[ T = 3/ w \times E \times I \times \cos\omega \]

8.9 starting torque and maximum torque at standstill condition

8.9.1 starting torque

At standstill condition
The torque produced by three phase induction motor depends upon the following three factors: Firstly the magnitude of rotor current, secondly the flux which interact with the rotor of three phase induction motor and is responsible for producing emf in the rotor part of induction motor, lastly the power factor of rotor of the three phase induction motor. Combining all these factors together we get the equation of torque as-

\[ T \propto \phi I_2 \cos \theta_2 \]

Where, T is the torque produced by induction motor,

\( \phi \) is flux responsible of producing induced emf,

\( I_2 \) is rotor current,

\( \cos \theta_2 \) is the power factor of rotor circuit.

The flux \( \phi \) produced by the stator is proportional to stator emf \( E_1 \).

i.e \( \phi \propto E_1 \)

We know that transformation ratio \( K \) is defined as the ratio of secondary voltage (rotor voltage) to that of primary voltage (stator voltage).

\[ K = \frac{E_2}{E_1} \]

or, \( K = \frac{E_2}{\phi} \)

or, \( E_2 = \phi \)
Rotor current  \( I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \)

**power factor** \( = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \)

The torque equation is \( T = \frac{3}{w} E_2 I_2 \cos \phi_2 \)

where

\( I_2 = \) rotor current at standstill

\( E_2 = \) rotor E.m.f. at standstill

\( \cos \phi_2 = \) rotor p.f. at standstill

so \( T_s = \frac{3}{w} E_2 \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \)

where \( \frac{3}{w} \) constant so can be write as \( K \)

\( T_s = K E_2 \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \)

\( T_s = K E_2^2 \frac{1}{\sqrt{R_2^2 + X_2^2}} \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \)

\( T_s = K E_2^2 \frac{R_2}{R_2^2 + X_2^2} \)

Where voltage supply of stator is constant, so the torque can be write as

\( T_s = K_1 \frac{R_2}{R_2^2 + X_2^2} \)

Where \( K_1 = K E_2^2 \)

It is clear that the magnitude of starting torque would depend upon the relative values of \( R_2 \) and \( X_2 \) i.e., rotor resistance/phase and standstill rotor reactance/phase.

\( T_s = \frac{3/2 \pi N_s E_2^2}{R_2^2 + X_2^2} \)
8.9.2 maximum torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

\[ T_s = K_1 \* \frac{R_2}{R_2^2 + X_2^2} \]

Differentiating eq. (i) w.r.t. \( R_2 \) and equating the result to zero, we get,

\[ \frac{dT_s}{dR_2} = \frac{K_1 (R_2^2 + X_2^2) - K_1 R_2 (2R_2)}{(R_2^2 + X_2^2)^2} = 0 \]

\[ K_1 (R_2^2 + X_2^2) - K_1 R_2 (2R_2) = 1 \]

\[ K_1 (R_2^2 + X_2^2) = K_1 R_2 (2R_2) \]

\[ (R_2^2 + X_2^2) = 2R_2^2 \]

\[ 2R_2^2 - R_2^2 = X_2^2 \]

\[ R_2^2 = X_2^2 \]

\[ R_2 = X_2 \]

8.10 starting torque and maximum torque at operation condition

8.10.1 starting torque at operation condition

Let the rotor at standstill have per phase induced e.m.f. \( E_2 \), reactance \( X_2 \) and resistance \( R_2 \). Then under running conditions at slip \( s \), as shown in Figure 8.15
Rotor current $I_2$ is defined as the ratio of rotor induced emf under running condition, $sE_2$ to total impedance, $Z_2$ of rotor side,

Rotor voltage $E_2 = sE_2$

Rotor reactance $X_2 = sX_2$

Rotor current $I_2 = \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$

Power factor $(\cos\phi_2) = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$

$Ts = 3/w \cdot E_2 \cdot \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$

$Ts = K_1 \cdot \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$

$Ts = K_2 \cdot \frac{sR_2}{R_2^2 + (sX_2)^2}$

where $K_2$ is another constant.

It may be seen that running torque is:
(i) directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.

(ii) directly proportional to square of supply voltage.

8.10.2 maximum torque at operation condition

\[ T_s = K_2 \times \frac{sR_2}{R_2^2 + (sX_2)^2} \]

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate above equation w.r.t. S and equate the result to zero i.e.,

\[ \frac{dT_s}{dS} = \frac{K_2 (R_2^2 + (s^2X_2^2) - K_2 2sR_2 (sR_2)}{(R_2^2 + (s^2X_2^2))^2} = 0 \]

\[ R_2 = sX_2 \]

Thus for maximum torque (Tm) under running conditions:

Rotor resistance/phase = Fractional slip × Standstill rotor reactance/phase

8.11 Torque-Slip Characteristics

The motor torque under running conditions is given by

\[ T_s = K_2 \times \frac{sR_2}{R_2^2 + (sX_2)^2} \]

If a curve is drawn between the torque and slip for a particular value of rotor resistance R2, the graph thus obtained is called torque-slip characteristic. Fig. (8.16) shows a family of torque-slip characteristics for a slip-range from s = 0 to s = 1 for various values of rotor resistance.
The following points may be noted carefully:

(i) At \( s = 0, T = 0 \) so that torque-slip curve starts from the origin.

(ii) At normal speed, slip is small so that \( s \times 2 \) is negligible as compared to \( R_2 \).

\[
\therefore T \propto \frac{s}{R_2}
\]

\[ T \propto s \text{ ... as } R_2 \text{ is constant} \]

Hence torque slip curve is a straight line from zero slip to a slip that corresponds to full-load.

(iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at \( s = R_2/X_2 \). This maximum torque in an induction motor is called pull-out torque or break-down torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.

(iv) To maximum torque, the term \( s^2 X^2 \) increases very rapidly so that \( R_2^2 \) may be neglected as compared

\[ s^2 X^2 \]

\[
\therefore T \propto \frac{s}{s^2 X^2}
\]
Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.

(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.

8.12 Speed control of three phase induction motor

8.12.1 Speed Control from Stator Side

1. \( V/f \) control or frequency control - Whenever three phase supply is given to three phase induction motor rotating magnetic field is produced which rotates at synchronous speed given by

\[
N_s = \frac{120f}{P}
\]

In three phase induction motor emf is induced by induction similar to that of transformer which is given by

\[
E \text{ or } V = 4.44\phi K.T.f \text{ or } \phi = \frac{V}{4.44KTf}
\]

Where \( K \) is the winding constant, \( T \) is the number of turns per phase and \( f \) is frequency. Now if we change frequency synchronous speed changes but with decrease in frequency flux will increase and this change in value of flux causes saturation of rotor and stator cores which will further cause increase in no load current of the motor. So, its important to maintain flux, \( \phi \) constant and it is only possible if we change voltage. i.e if we decrease frequency flux increases but at the same time if we decrease voltage flux will also decrease causing no change in flux and hence it remains constant. So, here we are keeping the ratio

\[
T \propto \frac{1}{s} \text{ ... as } X_2 \text{ is constant}
\]
of V/ f as constant. Hence its name is V/ f method. For controlling the speed of three phase induction motor by V/ f method we have to supply variable voltage and frequency which is easily obtained by using converter and inverter set.

2. Controlling supply voltage: The torque produced by running three phase induction motor is given by

\[ T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \]

In low slip region \((sX)^2\) is very very small as compared to \(R_2\). So, it can be neglected. So torque becomes

\[ T \propto \frac{sE_2^2}{R_2} \]

Since rotor resistance, \(R_2\) is constant so the equation of torque further reduces to

\[ T \propto sE_2^2 \]

We know that rotor induced emf \(E_2 \propto V\). So, \(T \propto sV^2\). From the equation above it is clear that if we decrease supply voltage torque will also decrease. But for supplying the same load, the torque must remains the same and it is only possible if we increase the slip and if the slip increases the motor will run at reduced speed. This method of speed control is rarely used because small change in speed requires large reduction in voltage, and hence the current drawn by motor increases, which cause over heating of induction motor.

3. Changing the number of stator poles: The stator poles can be changed by two methods

   i- Multiple stator winding method.

Multiple stator winding method – In this method of speed control of three phase induction motor, the stator is provided by two separate winding. These two stator
windings are electrically isolated from each other and are wound for two different pole numbers. Using switching arrangement, at a time, supply is given to one winding only and hence speed control is possible. Disadvantages of this method is that the smooth speed control is not possible. This method is more costly and less efficient as two different stator winding are required. This method of speed control can only be applied for squirrel cage motor.

ii- Pole amplitude modulation method (PAM)

Pole amplitude modulation method (PAM) – In this method of speed control of three phase induction motor the original sinusoidal mmf wave is modulated by another sinusoidal mmf wave having different number of poles.

Let \( f_1(\theta) \) be the original mmf wave of induction motor whose speed is to be controlled.

\( f_2(\theta) \) be the modulation mmf wave.

\( P_1 \) be the number of poles of induction motor whose speed is to be controlled.

\( P_2 \) be the number of poles of modulation wave.

\[
f_1(\theta) = F_1 \sin\left(\frac{P_1\theta}{2}\right)
\]

\[
f_2(\theta) = F_2 \sin\left(\frac{P_2\theta}{2}\right)
\]

After modulation resultant mmf wave

\[
F_r(\theta) = F_1 F_2 \sin\left(\frac{P_1\theta}{2}\right) \sin\left(\frac{P_2\theta}{2}\right)
\]

Apply formulae for \( 2 \sin A \sin B = \cos \frac{A - B}{2} - \cos \frac{A + B}{2} \)

So we get, resultant mmf wave

\[
F_r(\theta) = F_1 F_2 \frac{\cos \left(\frac{P_1 - P_2}{2}\right) - \cos \left(\frac{P_1 + P_2}{2}\right)\theta}{2}
\]
Therefore the resultant mmf wave will have two different number of poles

\[ i.e \ P_{11} = P_1 - P_2 \text{ and } P_{12} = P_1 + P_2 \]

Therefore by changing the number of poles we can easily change the speed of three phase induction motor.

4. Adding rheostat in the stator circuit - In this method of speed control of three phase induction motor rheostat is added in the stator circuit due to this voltage gets dropped. In case of three phase induction motor torque produced is given by \( T \propto sV_2^2 \). If we decrease supply voltage torque will also decrease. But for supplying the same load, the torque must remains the same and it is only possible if we increase the slip and if the slip increase motor will run reduced speed.

8.12.2 Speed Control from Rotor Side

1. Adding external resistance on rotor side – In this method of speed control of three phase induction motor external resistance are added on rotor side. The equation of torque for three phase induction motor is

\[ T \propto \frac{sE_2^2R_2}{R_2^2 + (sX)^2} \]

The three phase induction motor operates in low slip region. In low slip region term \((sX)^2\) becomes very very small as compared to \(R_2\). So, it can be neglected. and also \(E_2\) is constant. So the equation of torque after simplification becomes,

\[ T \propto \frac{s}{R_2} \]

Now if we increase rotor resistance, \(R_2\) torque decreases but to supply the same load torque must remains constant. So, we increase slip, which will further results in decrease in rotor speed. Thus by adding additional resistance in rotor circuit we can decrease the speed of three phase induction motor.
The main advantage of this method is that with addition of external resistance starting torque increases but this method of speed control of three phase induction motor also suffers from some disadvantages:

i- The speed above the normal value is not possible.

ii- Large speed change requires large value of resistance and if such large value of resistance is added in the circuit it will cause large copper loss and hence reduction in efficiency.

iii- Presence of resistance causes more losses.

iv- This method cannot be used for squirrel cage induction motor.

2. Cascade control method – In this method of speed control of three phase induction motor, the two three phase induction motor are connected on common shaft and hence called cascaded motor. One motor is the called the main motor and another motor is called the auxiliary motor. The three phase supply is given to the stator of the main motor while the auxiliary motor is derived at a slip frequency from the slip ring of main motor.

Let $N_{S1}$ be the synchronous speed of main motor.

$N_{S2}$ be the synchronous speed of auxiliary motor.

$P_1$ be the number of poles of the main motor.

$P_2$ be the number of poles of the auxiliary motor.

F is the supply frequency.

$F_1$ is the frequency of rotor induced emf of main motor.

N is the speed of set and it remains same for both the main and auxiliary motor as both the motors are mounted on common shaft.

\[ S_1 = \frac{N_{S1} - N}{N_{S1}} \]

\[ F_1 = S_1 F \]

The auxiliary motor is supplied with same frequency as the main motor i.e

\[ F_1 = F_2 \]

\[ N_{S2} = \frac{120F_2}{P_2} = \frac{120F_1}{F_2} \]

\[ N_{S2} = \frac{120S_1 F}{P_2} \]
Now put the value of
\[ S_1 = \frac{N_{S_1} - N}{N_{S_1}} \]

We get, \[ N_{S_2} = \frac{120F(N_{S_1} - N)}{P_2N_{S_1}} \]

Now at no load, the speed of auxiliary rotor is almost same as its synchronous speed i.e \( N = N_{S_2} \)

\[ N = \frac{120F(N_{S_1} - N)}{P_2N_{S_1}} \]

Now rearrange the above equation and find out the value of \( N \), we get,

\[ N = \frac{120F}{P_1 - P_2} \]

This cascaded set of two motors will now run at new speed having number of poles \((P_1 + P_2)\). In the above method the torque produced by the main and auxiliary motor will act in same direction, resulting in number of poles \((P_1 + P_2)\). Such type of cascading is called cumulative cascading. There is one more type of cascading in which the torque produced by the main motor is in opposite direction to that of auxiliary motor. Such type of cascading is called differential cascading; resulting in speed corresponds to number of poles \((P_1 - P_2)\).

In this method of speed control of three phase induction motor, four different speeds can be obtained

i- When only main induction motor work, having speed corresponds to 
\[ N_{S_1} = 120 F / P_1. \]

ii- When only auxiliary induction motor work, having speed corresponds to 
\[ N_{S_2} = 120 F / P_2. \]

iii- When cumulative cascading is done, then the complete set runs at a speed of 
\[ N = 120F / (P_1 + P_2). \]

iv- When differential cascading is done, then the complete set runs at a speed of 
\[ N = 120F / (P_1 - P_2). \]

3. Injecting slip frequency emf into rotor side - when the speed control of three phase induction motor is done by adding resistance in rotor circuit, some part of
power called, the slip power is lost as $I^2R$ losses. Therefore the efficiency of three phase induction motor is reduced by this method of speed control. This slip power loss can be recovered and supplied back in order to improve the overall efficiency of three phase induction motor and this scheme of recovering the power is called slip power recovery scheme and this is done by connecting an external source of emf of slip frequency to the rotor circuit. The injected emf can either oppose the rotor induced emf or aids the rotor induced emf. If it oppose the rotor induced emf, the total rotor resistance increases and hence speed decreases and if the injected emf aids the main rotor emf the total resistance decreases and hence speed increases. Therefore by injecting induced emf in rotor circuit the speed can be easily controlled. The main advantage of this type of speed control of three phase induction motor is that wide range of speed control is possible whether its above normal or below normal speed.

8.13 Starting of three phase induction motor

A 3-phase induction motor is theoretically self starting. The stator of an induction motor consists of 3-phase windings, which when connected to a 3-phase supply creates a rotating magnetic field. This will link and cut the rotor conductors which in turn will induce a current in the rotor conductors and create a rotor magnetic field. The magnetic field created by the rotor will interact with the rotating magnetic field in the stator and produce rotation. Therefore, 3-phase induction motors employ a starting method not to provide a starting torque at the rotor, but because of the following reasons;

1) Reduce heavy starting currents and prevent motor from overheating.

2) Provide overload and no-voltage protection.

There are many methods in use to start 3-phase induction motors. Some of the common methods are;

i. Direct On-Line Starter (DOL)
ii. Star-Delta Starter  
iii. Auto Transformer Starter  
iv. Rotor Impedance Starter  

8.13.1 Direct On-Line Starter (DOL)  

The Direct On-Line (DOL) starter is the simplest and the most inexpensive of all starting methods and is usually used for squirrel cage induction motors. It directly connects the contacts of the motor to the full supply voltage. The starting current is very large, normally 6 to 8 times the rated current. The starting torque is likely to be 0.75 to 2 times the full load torque. In order to avoid excessive voltage drops in the supply line due to high starting currents, the DOL starter is used only for motors with a rating of less than 5KW. There are safety mechanisms inside the DOL starter which provides protection to the motor as well as the operator of the motor. The power and control circuits of induction motor with DOL starter and the real picture of contactor are shown in Figure 8.17 and Figure 8.18 respectively.
Figure 8.17 power and control circuits of three phase induction motor with DOL starter.

Figure 8.18 real picture of contactor with overload.

The DOL starter consists of a coil operated contactor K1M controlled by start and stop push buttons. On pressing the start push button S1, the contactor coil K1M is energized from line L1. The three mains contacts (1-2), (3-4), and (5-6) in fig. (8.17) are closed. The motor is thus connected to the supply. When the stop push button S2 is pressed, the supply through the contactor K1M is disconnected. Since the K1M is de-energized, the main contacts (1-2), (3-4), and (5-6) are opened. The supply to motor is disconnected and the motor stops.

8.13.2 Star-Delta Starter

The star delta starting is a very common type of starter and extensively used, compared to the other types of the starters. This method uses reduced supply voltage in starting. Figure(8.19) shows the connection of a 3 phase induction motor with a star – delta starter. The method achieved low starting current by first connecting the stator winding in star configuration, and then after the motor reaches a certain speed, throw switch changes the winding arrangements from star to delta configuration. By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the
windings connected in delta. At the time of starting when the stator windings are start connected, each stator phase gets voltage $VL / \sqrt{3}$, where $VL$ is the line voltage. Since the torque developed by an induction motor is proportional to the square of the applied voltage, star–delta starting reduced the starting torque to one – third that obtainable by direct delta starting. Figure 8.20 shows real picture of star delta starter.

Figure 8.19 power and control circuit of three phase induction motor with star delta starter.
8.13.3 Auto Transformer Starter

The operation principle of auto transformer method is similar to the star delta starter method. The starting current is limited by (using a three phase auto transformer) reduce the initial stator applied voltage. The auto transformer starter is more expensive, more complicated in operation and bulkier in construction when compared with the star – delta starter method. But an auto transformer starter is suitable for both star and delta connected motors, and the starting current and torque can be adjusted to a desired value by taking the correct tapping from the auto transformer. When the star delta method is considered, voltage can be adjusted only by factor of $1/\sqrt{3}$. Figure (8.21) shows the connection of a 3phase induction motor with auto transformer starter. It can brief operation of auto transformer as :

1. Operated by a two position switch i.e. manually / automatically using a timer to change over from start to run position.
2. In starting position supply is connected to stator windings through an auto-transformer which reduces applied voltage to 50, 60, and 70% of normal value depending on tapping used.
3. Reduced voltage reduces current in motor windings with 50% tapping used motor current is halved and supply current will be half of the motor current. Thus starting current taken from supply will only be 25% of the taken by DOL starter.

4. For an induction motor, torque $T$ is developed by $V^2$, thus on 50% tapping, torque at starting is only $(0.5V)^2$ of the obtained by DOL starting. Hence 25% torque is produced.

5. Starters used in larger industries, it is larger in size and expensive.

6. Switching from start to run positions causing transient current, which can be greater in value than those obtained by DOL starting.

Figure 8.21 power and control circuit of three phase induction motor with auto transformer starter.

8.13.4 Rotor Impedance Starter

This method allows external resistance to be connected to the rotor through slip rings and brushes. Initially, the rotor resistance is set to maximum and is then gradually decreased as the motor speed increases, until it becomes zero. The rotor impedance starting mechanism is usually very bulky and expensive when compared with other
methods. It also has very high maintenance costs. Also, a considerable amount of heat is generated through the resistors when current runs through them. The starting frequency is also limited in this method. However, the rotor impedance method allows the motor to be started while on load. Figure (8.22) shows the connection of a 3-phase induction motor with rotor resistance starter.

Figure 8.22 induction motor with rotor resistance starter.

This will decrease the starting current, increases the starting torque and also improves the power factor. The circuit diagram is shown below: In the circuit diagram, the three slip rings shown are connected to the rotor terminals of the wound rotor motor. At the time of starting of the motor, the entire external resistance is added in the rotor circuit. Then the external rotor resistance is decreased in steps as the rotor speeds up, however the motor torque remain maximum during the acceleration period of the motor. Under normal condition when the motor develops load torque the external resistance is removed.
8.14 Equivalent circuit of three phase induction motor

From the preceding, we can utilise the equivalent circuit of a transformer to model an induction motor as shown in Figure 8.23.

In the equivalent circuit $R_1$ represents, the resistance of the stator winding and $X_1$ the stator leakage reactance (flux that does not link with the air gap and rotor). Magnetising reactance required to cross the air gap is represented by $X_m$ and core losses (hysteresis and eddy current) by $R_c$. An ideal transformer of $N_1$ and $N_2$ turns respectively represents the air gap. For the rotor side, the induced emf is affected by the slip (as the rotor gains speed, slip reduces and less emf is induced). The rotor resistance and reactance are represented by $R_2$ and $X_2$; with $X_2$ being dependant on the frequency of the inductor rotor emfs.

$$EI = 4.44 f_1 N \Phi$$

At standstill $E_2 = 4.44 f_1 N \Phi$

At running $E_2s = 4.44 f_2 N \Phi = 4.44 sf_1 N \Phi = sE_2$

At standstill Rotor reactance $= X_2 = WL = 2\pi f_1 L$

At running Rotor reactance $= X_2 = WL = 2\pi f_2 L = 2\pi sf_1 L = sX_2$

The rotor circuit, the current $I_2$ is given by:
\[ I_2 = \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \text{ divided by } s \]

Which can be written as

\[ I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2/s)^2 + (sX_2)^2}} \]

The above equality allows the equivalent circuit to be drawn as shown in figure 8.24:

**Simplified Equivalent Circuit**

The equivalent circuit shown above has removed the dependence on slip for determining the secondary voltage and frequency. Consequently, the circuit can be simplified by eliminating the ideal transformer and referring the rotor's resistance and reactance to the primary (denoted by ') as shown in figure 8.25.

![Figure 8.24](image-url)
The referred values are calculated by multiplying them by $k^2$, where $k$ is the effective stator/rotor turns ratio.

**Calculation of Motor Performance**
The simplified equivalent circuit allows us to calculate the operating parameters for an induction motor.

**Motor Current**
Once the equivalent circuit parameters are known, it is easy to calculate the motor current, by reducing the circuit to an equivalent impedance $Z_{eq}$, giving:

$$I_1 = \frac{V_1}{Z_{eq}}$$

**Motor Power**
Input power ($P_{in}$) = $\sqrt{3} V I L \cos \phi$

Power of Stator copper loss ($P_{SCL}$) = $3 \times I1^2 R1$

Power of Core loss ($P_{CORE}$) = $3 \times \frac{E1^2}{R1}$

So the air gap power ($P_{AG}$) = $P_{in} - P_{SCL} - P_{CORE}$

From rotor equivalent circuit the air gap power can consume only the resistance $\frac{R2}{s}$

So $P_{AG} = 3 \times I2^2 \frac{R2^2}{s}$

Power of rotor copper loss ($P_{RCL}$) = $3 \times I2^2 R2$

The power converted ($P_{con}$) or mechanical developed power ($P_{mech}$) = $P_{AG} \cdot P_{RCL}$
\[ I_2^2 \frac{R_2^2}{s} - 3 \cdot I_2^2 \cdot R_2 = 3 \cdot I_2^2 \cdot R_2 \left( \frac{1}{s} - 1 \right) = 3 \cdot I_2^2 \cdot R_2 \left( \frac{1-s}{s} \right) = P_{RCL} \left( \frac{1}{s} - 1 \right) \]

Or \( P_{mech} = P_{in} - P_{SCL} - P_{CORE} - P_{RCL} \)

Power output (\( P_{out} \)) or power shift = \( P_{con} \), power friction & windage – power stray

\[ P_{in} = \sqrt{3} V_f I_L \cos \theta \]

Motor Torque

The induced torque (\( T_{ind} \)) or developed torque in machine was define as the torque generated by electrical to mechanical power conversion given by:

\[ T_{ind} = \frac{P_{con}}{W} \]

8.15 Examples:

1. A 12 pole, 3 phase alternator is coupled to an engine running at 500 rpm. It supplies an Induction Motor which has a full load speed of 1440 rpm. Find the percentage slope and the number of poles of the motor.
Solution: \( N_A = \text{synchronous speed of the alternator} \)

\[
\frac{P N_A}{12 \times 500} = \frac{F}{120} = \frac{50 \text{ Hz (from alternator data)}}{120} = 50 \text{ Hz}
\]

When the supply frequency is 50 Hz, the synchronous speed can be 750 rpm, 1500 rpm, 3000 rpm etc., since the actual speed is 1440 rpm and the slip is always less than 5% the synchronous speed of the Induction motor is 1500 rpm.

\[
\frac{N_s - N}{N_s} = \frac{1500 - 1440}{1500} = 0.04 \text{ OR } 4\%
\]

\[
\frac{N_s}{120f} = \frac{120 \times 50}{120} = 1500 \text{ rpm}
\]

\( P = 4 \)

2. A 6 pole induction motor is supplied by a 10 pole alternator, which is driven at 600 rpm. If the induction motor is running at 970 rpm, determine its percentage slip.

Solution:

\[
\frac{P N_A}{10 \times 600} = \frac{f}{120} = \frac{50 \text{ Hz (from alternator data)}}{120} = 50 \text{ Hz}
\]

Synchronous speed of the induction motor

\[
\frac{120f}{120 \times 50} = \frac{120 \times 50}{120} = 1000 \text{ rpm}
\]

From I.M. data:

\[
\frac{N_s - N}{P} \times 100 = \frac{1000 - 970}{1000} = 3\%
\]

3. A 12 pole, 3 phase alternator is driven by a 440V, 3 phase, 6 pole Induction Motor running at a slip of 3%. Find frequency of the EMF generated by the alternator.
Solution:

For induction motor: \( N_s = \frac{120f}{P} \times \frac{120 \times 50}{6} = 1000\text{rpm} \)

\( N = (1 - s) N_s = (1 - 0.03) 1000 = 970\text{rpm} \)

As the alternator is driven by the Induction motor, the alternator runs at 970 r.p.m.

For induction motor: \( f = \frac{PN}{120} \times \frac{12 \times 970}{120} = 97\text{Hz} \)

4. A three phase 4 pole, 440 V, 50Hz induction motor runs with a slip of 4%. Find the rotor speed and frequency of the rotor current.

Solution:

For induction motor: \( N_s = \frac{120f}{P} \times \frac{120 \times 50}{4} = 1500\text{rpm} \)

\( N_s - N = 1500 - N \)

solution: \( S = \frac{N_s - N}{N_s} \times \frac{i.e, 0.04}{1500} = \frac{N = 1440\text{rpm}}{N} \)

\( f_r = sf - 0.04 \times 50 = 2\text{Hz} \)

5. A 3 phase, 50Hz 6 pole induction motor has a full load percentage slip of 3%. Find
   - Synchronous speed and
   - Actual Speed

Solution:

\( N_s = \frac{120f}{P} \times \frac{120 \times 50}{6} = 1000\text{rpm} \)

\( N_s - N = 100 - N \)

\( S = \frac{N_s - N}{N_s} \times \frac{i.e, 0.03}{1000} = \frac{N = 970\text{rpm}}{N} \)

6. A 3 phase induction motor has 6 poles and runs at 960 RPM on full load. It is supplied from an alternator having 4 poles and running at 1500 RPM. Calculate the full load slip and the frequency of the rotor currents of the induction motor.
Solution:

\[
\text{PN} = 4 \times 1500
\]

\[
f = \frac{4}{120} = \frac{1500}{120} = 50\text{Hz (from alternator data)}
\]

for Induction motor

\[
120f = \frac{120 \times 50}{P} = 6
\]

\[
N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000\text{rpm}
\]

\[
N_s - N = 1000 - 960
\]

\[
S = \frac{1000 - 960}{N} = \frac{1000 - 960}{1000} = 0.04 \text{ or } 4\%
\]

\[
f_r = sf - 0.04 \times 50 = 2\text{Hz}
\]

7. The frequency of the e.m.f in the stator of a 4-pole induction motor is 50 Hz and that of the rotor is 1\(\frac{1}{2}\)Hz. What is the slip and at what speed is the motor running?

Solution:

Given \(P = 4\)

\(f = 50\text{Hz}\)

\(f_r = 1.5\text{Hz}\)

- To calculate slip (s)

\[
f_r = sf
\]

\[
1.5 = S \times 50
\]

we have \(s = \frac{1.5}{50} = 0.03\),

\(s = 0.03\),

\(s = \text{or } 3\%\)

- To calculate the speed of the motor \((N)\)

We have

\[
120f = \frac{N_s}{P}
\]

\[
120 \times 50 = \frac{N_s}{4} = 1500\text{rpm}
\]

We also have

\(N = N_s (1 - s)\)
\[ N = N_s \times 1500(1 - 0.03) \]
\[ N = 1455 \text{rpm} \]

8. A 3-phase, 60Hz induction motor has a slip of 3% at full load. Find the synchronous speed, the full-load speed and the frequency of rotor current at full load.

Solution:

Given \( P = 6 \)

\( f = 60 \text{Hz} \)
\( s = 3\% = 0.03 \)

- To find the synchronous speed (\( N_s \))
  
  We have
  
  \[ \frac{120f}{P} = \frac{120 \times 50}{6} = \frac{6000}{6} = 1000 \text{rpm} \]

- To calculate the full load speed (\( N \))
  
  We have
  
  \[ N = N_s(1 - s) = 1200(1 - 0.03) = 1164 \text{rpm} \]

- To calculate the frequency of the rotor current (\( f_r \))
  
  We have
  
  \[ f_r = sf = 0.03 \times 60 = 1.8 \text{Hz} \]

9. A 6-pole alternator running at 600 rpm supplies a 3-phase, 4-pole induction motor. If the induction motor induced e.m.f makes 2 alternations per second, find the speed of the motor.

Solution:

- Alternator

  \( P = 6 \)

  \( N_s = 600 \)

  We have the frequency of induced e.m.f of an alternator given by

  \[ \frac{PN_s}{120} = \frac{6 \times 600}{120} = 30 \text{Hz} \]

  Hence, induction motor receives the supply at 30Hz frequency

- Induction motor

  It is given that the rotor induced e.m.f makes two alternations per second i.e. 1.0 cycle per second
\[ f_r = 1.0 \text{Hz} \]
\[ f = sf \]

We have

\[
\begin{align*}
    \frac{f}{s} &= 1.0 \\
    s &= \frac{f}{s} = \frac{1.0}{s} = 30 \text{Hz}
\end{align*}
\]

The speed of the rotating magnetic field is given by

\[
N_s = \frac{120f}{P_m} \quad \text{Where } P_m = \text{Number of poles of induction motor}
\]

\[
N_s = \frac{120 \times 30}{10} = \frac{3600}{10} = 360 \text{rpm}
\]

The speed of the induction motor (N) is given by

\[
N = N_s (1 - s) = 900 (1 - 0.03)
\]

\[
N = 870 \text{rpm}
\]

10. A 10-pole induction motor is supplied by a 6-pole alternator, which is driven at 1200 rpm. If the motor runs with a slip of 3, what is the speed of the induction motor?

Solution

- **Alternator**
  - \( P = 6 \text{ pole} \)
  - \( N_s = 1200 \text{rpm} \)
  - Therefore, the frequency of the e.m.f generated is given by
    \[
    f = \frac{PN_s}{120} = \frac{6 \times 1200}{120} = 60 \text{Hz}
    \]
  - Hence, the induction motor is supplied at 60 Hz frequency

- **Induction motor**
  - Supply frequency = \( f = 60 \text{Hz} \)
  - Slip = \( S = 3\% \)
  - Stator poles = \( P_m = 10 \)
  - Speed of the rotating magnetic field,
    \[
    N_s = \frac{120f}{P_m} = \frac{120 \times 60}{10} = 720 \text{rpm}
    \]
  - \( N_s = 720 \text{rpm} \)
  - Speed of the motor
N = Ns(1 - s) = 900 (1 - 0.03)  
N = 698.4rpm

11. A 3-phase induction motor has 6-poles and runs at 960r.p.m. on full load. It is supplied from an alternator having 4-poles and running at 1500 r.p.m. calculate the full load slip of the motor.

Solution

- Alternator  
P = 4 poles  
Ns = 1500rpm  
The frequency generated e.m.f is given by  
\[ f = \frac{PNs}{120} = \frac{4 \times 1500}{120} \] 
f = 50Hz  
Hence, the induction motor is supplied at 50 Hz

- Induction motor  
P = 4 poles  
f = 50Hz  
N = 960rpm  
The speed of the rotating magnetic field is given by  
\[ N_s = \frac{120f}{P_m} = \frac{120 \times 50}{6} \] 
Ns = 1000rpm  
We have slip of an induction motor given by  
\[ S = \frac{N_s - N}{N_s} = \frac{1000 - 960}{1000} = 0.04 \] 
s = 4%

12. A 4-pole, 30hp, 3-phase 400 volts, 50Hz induction motor operates at an efficiency of 0.85 with a power factor of 0.75(lag). Calculate the current drawn by the induction motor from the mains.

Solution  
Given P = 4  
V = 400  
\[ \eta = 0.85 \]  
\[ \cos \varphi = 0.75 \]  
Output= 30 hp  
\[ = 22.06538 \text{ Kw} \ (1 \text{metric hp}=735.5 \text{ watts}) \]  
We have,
output
\[ \eta = \frac{\text{output}}{\text{input}} \]

input = 22.065
\[ \eta = \frac{22.065}{\text{input}} = 0.85 \]

input = 25.96KW

But, for a 3-phase induction motor circuit, the power input is also given by the expression
\[ P = \sqrt{3} V_L I_L \cos \Phi \]

\[ P = \frac{25.96 \times 1000}{\sqrt{3} V_L I_L \cos \Phi} = \frac{\sqrt{3} V_L I_L \cos \Phi}{\sqrt{3} \times 400 \times 0.75} \]
\[ I_L = 49.94 \text{ Amperes} \]

13. A 5 hp, 400V, 50Hz, 6-pole, 3-phase induction motor operating on full load draws a line current of 7 amperes at 0.866 power factor with 2% slip. Find the rotor speed.

Solution

Given \( P = 6 \)
\( s = 2\% \)
\( = 0.02 \)
\( \cos \Phi = 0.866 \)
\( f = 50 \text{ Hz} \)
output = 5 hp = 5 \times 735.5 \text{ watts}
output = 3.677 KW
\( I_L = 7 \text{ Amperes} \)

To find the rotor speed:

Speed of the stator magnetic field is given by
\[ N_s = \frac{120f}{P_m} = \frac{120 \times 50}{6} \]
\[ N_s = 1000 \text{ rpm} \]

Speed of the rotor = \( N = N_s (1-s) \)
\[ = 1000(1-0.02) \]
\[ N = 980 \text{ rpm} \]
14. A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1% at no-load and 3% at full-load.

Find:
1. Synchronous speed,
2. No-load speed
3. Full-load speed,
4. Frequency of rotor current at standstill, and
5. Frequency of rotor current at full-load

Solution. Number of poles, $p = 6$

No-load slip, $s_0 = 1\%$
Full-load slip, $s_f = 3\%$

6. Synchronous speed,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{60} = 1000 \text{ rpm}$$

7. No-load speed $N_0$

We know that

$$\frac{N_r - N}{N_s} = s \quad \text{or} \quad N = N_s(1 - s)$$

8. Full-load speed

9. Frequency of rotor current at standstill, $f_r$

$$s = 1$$

$$f_r = s \times f = \frac{1 \times 50}{100} = 50 \text{ Hz} \quad \text{(Ans)}$$

10. Frequency of rotor current at full-load, $f_r = ?$

$$f_r = s \times f = \frac{1 \times 1455}{100} = 14.55 \text{ Hz} \quad \text{(Ans)}$$

A 3-phase, 12-pole alternator is coupled to an engine running at 500 rpm. The alternator supplies an induction motor which has a full-load speed of 1455 rpm. Find the slip and number of poles of the motor.

Solution.

Number of poles of the alternator, $p_a = 12$

Speed of the engine, $N_e = 500 \text{ rpm}$

Full-load speed of the induction motor, $N_m = 1455 \text{ rpm}$

Slip, $s = ?$

Number of poles of the induction motor, $P_m = ?$

Supply frequency,
\[
N_aP_a = \frac{500 \times 12}{120} = 50Hz
\]

When the supply frequency is 50Hz, the synchronous speed can be 3000, 1500, 1000, 750 rpm etc. since the full-load speed is 1455rpm and the full-load slip is always less than 4%, the synchronous speed is 1500rpm.

\[
S = \frac{N_s - N_{1500}}{1500 - 1455} = \frac{1500 - 1455}{0.03} = 0.03 \text{ or } 3\% \ (\text{Ans})
\]

Also,
\[
N_s = \frac{120f}{P_m} = \frac{120 \times 50}{\frac{120}{1500}} = 4 \text{ poles}
\]

Hence, number of motor poles = 4. (Ans)

A 4-pole, 50 Hz induction motor at no-load (NNL) has a slip of 2%. When operated at full load the slip increases to 3%. Find the change in speed of the motor from no-load to full load.

Solution:

\[
N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\text{rpm}
\]

no-load speed \(N_{NL} = N_s (1 - S_{NL})\)

= 1500 (1-0.02) = 1470rpm

full load speed \(N = N_s (1 - S_{FL})\)

= 1500 (1-0.03) = 1455rpm

change in speed from no-load to full load

\(N_{NL} - N_{FL} = 1470 - 1455 = 15\text{rpm.}\)
17 The rotor resistance and standstill reactance per phase of a 3–phase slip–ring induction motor are 0.05Ω and 0.1Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to give maximum torque at starting?

Solution

Let external resistance per phase added to the rotor circuit be $r$ ohms. Rotor resistance per phase, $R_2 = (0.05 + r)$. The starting torque should be maximum when

$$R_2 = X_{20}$$

$$0.05 + r = 0.1$$

$$\therefore r = 0.05\Omega$$

18 A 3-phase induction motor with rotor resistance $r$ phase equal to the standstill rotor reactance has a starting torque of 25 Nm. For negligible stator impedance and no load current, determine the starting torque in case the rotor circuit resistance per phase is (a) doubled, (b) halved.

Solution

$$R_2 = X_{20}$$

$$T_s = \frac{kR_2}{R_2^2 + X_{20}^2}$$

$$25 = \frac{kR_2}{R_2^2 + X_{20}^2}$$

$$\therefore k = 50R_2$$

(a) New rotor resistance = $2R_2$

$$\therefore T_s = \frac{k(2R_2)}{(2R_2)^2 + R_2^2} = \frac{50R_2 \cdot 2R_2}{5R_2^2} = 20\ Nm$$

(b) Rotor resistance = $\frac{1}{2}R_2$

$$\therefore T_s = \frac{k(\frac{1}{2}R_2)}{(\frac{1}{2}R_2)^2 + R_2^2} = \frac{50R_2 \cdot \frac{1}{2}R_2}{\frac{R_2^2}{4} + R_2^2} = 20\ Nm$$
19 A 6-pole, 3-phase, 50 Hz induction motor develops a maximum torque of 30Nm at 960 r.p.m. Determine the torque exerted by the motor at 5% slip. The rotor resistance per phase is 0.6Ω.

Solution:

\[ N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.} \]

Speed at maximum torque, \( N_m = 960 \text{ r.p.m.} \)

Slip at maximum speed

\[ s_M = \frac{N_s - N_m}{N_s} = \frac{1000 - 960}{1000} = 0.04 \]

Also

\[ s_M = \frac{R_2}{X_{2o}} \]

\[ X_{2o} = \frac{R_2}{s_M} = \frac{0.6}{0.04} = 15 \Omega \]

If \( T_{fs} \) is the torque at slip \( s \)

\[ \frac{T_{fs}}{T_M} = \frac{2ss_M}{s^2 + s_M^2} \]

Here \( s = 0.05 \), \( T_M = 30 \text{ Nm} \)

\[ T_{fs} = \frac{2 \times 0.05 \times 0.04}{(0.05)^2 + (0.04)^2} \times 30 = 29.27 \text{ Nm} \]
A 6 pole, 50 Hz, 3 phase induction motor running on full load develops a useful torque of 150 N.m at a rotor frequency of 1.5Hz. Calculate the shaft power output, if the mechanical n be 10 N.m, determine (a) rotor copper loss, (b) the input to the motor, and (c) the efficiency. The total stator loss is 700 watt. Neglect the power core loss.

**Solution:**

\[ N_s = \frac{120f_s}{p} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.} \]

\[ S = \frac{f_r}{f_s} = \frac{1.5}{50} = 0.03 \text{ or } 3\% \]

\[ N_r = (1 - S)N_s = (1 - 0.03) \times 1000 = 970 \text{ r.p.m.} \]

\[ \omega_r = 2\pi N_r = \frac{2\pi \times 970}{60} = 101.58 \text{ rad/s} \]

Shaft power output, \( P_o = T_o \omega_r \)

\[ = 150 \times 101.58 = 15.236 \text{ kw} \]
Mechanical power developed

\[ P_{md} = (150 + 10) \times 101.58 = 16.252 \text{ kw} \]

(a) Rotor copper loss \( P_{rc} = \left( \frac{S}{1-S} \right) P_{md} \)

\[ = \frac{0.03}{1 - 0.03} \times 16.252 = 0.5026 \text{ kw} \]

(b) Input to motor \( P_{t} = P_{md} + P_{rc} + P_{sc} \)

\[ = 16.252 + 0.5026 + 0.7 = 17.4546 \text{ kw} \]

(c) \textit{efficiency} \( = \frac{P_{o}}{P_{in}} = \frac{15.236}{17.4546} = 87.29\% \)

21: a 500V, 6-pole, 50Hz, 3-phase induction motor develops 20kw inclusive of mechanical losses when running at 995 r.p.m., the p.f. being 0.87. Calculate (a) the slip, (b) the rotor I2 R loss, (c) the total input if the stator loss is 1500 w, (d) line current, (e) the rotor current frequency.

Solution:

\[ N_{S} = \frac{120f_{s}}{P} = \frac{120 \times 50}{60} = 1000 \text{ r. p. m.} \]

\[ s = \frac{N_{S} - N_{r}}{N_{S}} = \frac{1000 - 995}{1000} = 0.005 \text{ pu} \]
Rotor on loss = slip * rotor power input

\[ P_{rc} = s(P_m + P_{rc}) \]

\[ P_{rc}(1 - s) = sP_m \]

\[ P_{rc} = \frac{sP_m}{(1 - s)} = \frac{0.005}{1 - 0.005} \times 20 \times 1000 = 100.5 \text{ W} \]

Total input to stator = rotor power input + stator loss

\[ Rotor\ input = \frac{1}{s} \times rotor\ Cu\ loss \]

\[ = \frac{1}{0.005} \times 100.5 = 20100\ W = 20.1\ KW \]

*Hence total input = 20.1 + 1.5 = 21.6KW*

\[ Line\ current = \frac{21600}{\sqrt{3} \times 500 \times 0.87} = 28.7A \]

\[ f_r = sf_s = 0.005 \times 50 = 0.25\ Hz \]

8.16 Tutorial problems
Tutorial 3

1. A 3 phase induction machine 373kW, 6 poles is connected to a 440V, 50 Hz, has a full load speed of 950 rpm. If the machine is comprised of 6 poles, calculate the frequency of the rotor current during full load.

2. Determine the synchronous speed of a six pole 460V 60 Hz induction motor if the frequency is reduced to 85% of its rated value.

3. A 4 pole induction machine is supplied by 60 Hz source and having 4% of full load slip. Calculate the rotor frequency during:
   (i) Starting
   (ii) Full load

4. A 3-phase induction motor, delta/star connection, 2 poles, 50 Hz is connected to a 410V source. The rotor speed is 2880 rpm and the windage and friction losses are 600 W. The equivalent circuit per phase referred to the stator circuit is:
   \[ R_s = 0.4 \Omega \quad X_s = 2 \Omega \]
   \[ R_r = 2 \Omega \quad R_m = 150 \Omega \]
   Calculate:
   (i) Input power
   (ii) Air-gap power
   (iii) Mechanical power
   (iv) Developed torque/torque induced
   (v) Efficiency

5. A 440V, 50Hz, 10 pole, delta/Y connected induction motor is rated at 100kW. The equivalent parameter for the motor are:
   \[ R_s = 0.08 \Omega / \text{phase} \quad R_m = 0.1 \Omega / \text{phase} \]
   \[ X_s = 0.3 \Omega / \text{phase} \quad X_m = 0.2 \Omega / \text{phase} \]
   \[ X_a = 5 \Omega / \text{phase} \quad R_a = \infty \]
   At full load condition, the friction and windage losses are 400W, the miscellaneous losses are 100W and the core losses are 1000W. The slip of the motor is 0.04.
   (i) Calculate the input power
   (ii) Calculate the stator copper loss
   (iii) Calculate the air gap power
   (iv) Calculate the converted power
   (v) Calculate the torque induced by the motor
   (vi) Calculate the load torque
   (vii) Calculate the starting torque
   (viii) Calculate the maximum torque and slip
   (ix) Calculate the efficiency of the motor
6. Squirrel cage and wound rotors are the two common types of rotor used in induction machines. Give four advantages of squirrel cage rotor.

7. A 4 pole induction machine is supplied by 50 Hz source and having 4% of full load slip. Find the rotor frequency during:
   (i) Starting
   (ii) Full load

8. A 3-phase, Y-connected, 50 Hz, 4 pair of poles, induction motor having 720 rpm full load speed. The motor is connected to a 415 V supply. The machine has the following impedances in ohms per phase referred to the stator circuit:
   \[ R_1 = 0.2 \, \Omega \quad X_1 = 2.0 \, \Omega \]
   \[ R_2 = 0.9 \, \Omega \quad X_2 = 4.0 \, \Omega \]
   \[ X_m = 60 \, \Omega \]
   If the total friction and windage losses are 200 W,
   (i) Find the slip, \( s \)
   (ii) Find the input power, \( P_i \)
   (iii) Find the air gap power, \( P_{ag} \)
   (iv) Find the mechanical power, \( P_m \)
   (v) Find the torque induced by the motor, \( \tau_{ind} \)
   (vi) Find the efficiency of the motor.

9. Induction machine is a common type of AC machine. State three weaknesses of the induction machine.

10. A 3-phase, delta-connected, 50 Hz, 2 pair of poles, induction motor having 1455 rpm full load speed. The motor is connected to a 415 V supply. The machine has the following impedances in ohms per phase referred to the stator circuit:
    \[ R_1 = 0.2 \, \Omega \quad X_1 = 0.6 \, \Omega \]
    \[ R_2 = 0.9 \, \Omega \quad X_2 = 0.4 \, \Omega \]
    \[ X_m = 20 \, \Omega \]
    If the total friction and windage losses are 1000 W, calculate:
    (i) Slip
    (ii) Input power, \( P_i \)
    (iii) Air gap power, \( P_{ag} \)
    (iv) Mechanical power, \( P_m \)
    (v) Torque induced by the motor, \( \tau_{ind} \)
    (vi) Efficiency of the motor

11. A 3-phase, Y-connected, 6 poles, 415 V, 50 Hz induction motor having a rotor speed 950 rpm. The input power is 100 kW at 0.85 power factor lagging. The copper and iron losses in the stator are 4 kW and the windage and friction losses are 4 kW. Determine the output power of the motor.